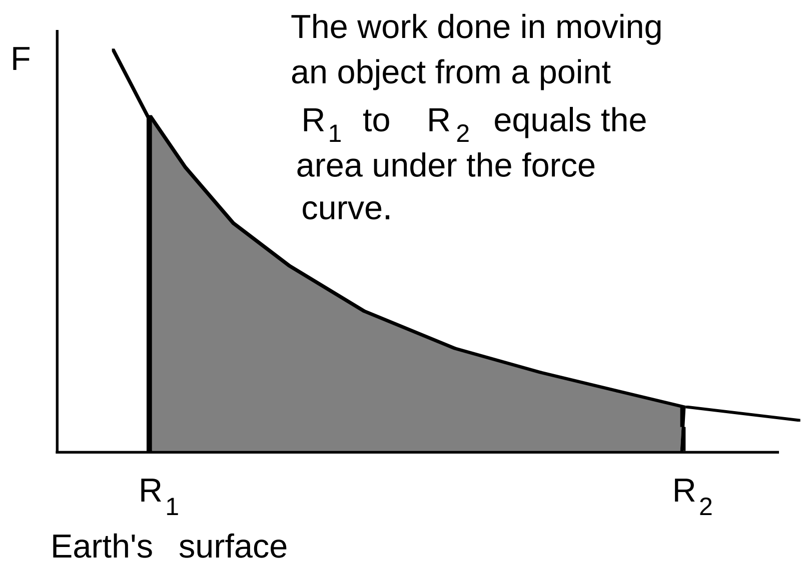
Gravitational Potential Energy : Notes-15

The gravitational potential energy (GPE) of a body near the Earth's surface is equal to mgh. The potential energy is assumed to be zero at ground level. This formula works well if h is small, because the force of gravity equals mg near the Earth's surface.

Satellites are not near the Earth's surface. The force of gravity decreases with increasing distance from the Earth's center. A new formula for the potential energy is needed.



The work done in moving an object of mass m, can be found by using the calculus. It is given by:

$$\text{Work} = GM_E m\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Since Work = ΔGPE = GPE_2 - GPE_1, we define the new gravitational potential energy for a mass m as:

$$G.P.E. = -\frac{GM_E m}{R}$$

R is the distance from the Earth's center. The potential energy of a mass m at a large distance from Earth's center is zero.

Conservation of Energy

The Law of Conservation of Energy states that: the total energy of a system remains constant, if there are no external forces. For a satellite moving in a gravitational field, we have:

$$\frac{1}{2}mV_1^2 - \frac{GMm}{R_1} = \frac{1}{2}mV_2^2 - \frac{GMm}{R_2}$$

We can use the above formula to find the escape velocity for a projectile from a planet. If the velocity is too small, the projectile will fall back to the ground. At a certain minimum speed, the projectile will escape the planet's gravity. Assume the velocity at large R_2 is zero. The escape velocity is V_1.

$$\frac{1}{2}mV_1^2 - \frac{GMm}{R_1} = 0$$

Solving for V_1, we have:

$$V_1 = \sqrt{\frac{2GM}{R_1}}$$

M is the mass of the planet and R_1 is the radius of the planet. V_1 is the minimum speed necessary to leave the planet.