Relativity : Notes/W.S.-50

There are several more relativistic equations that will be stated without proof. The proofs require the calculus which are beyond the scope of this work.

Einstein showed that the mass of an object increases with its velocity according to the equation:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The mass of a particle when it is at rest is m_o. This is called the **rest mass** of the particle.

The equation above shows that the mass of the particle increases with its velocity, and approaches infinity at the speed of light. Einstein said that nothing could travel faster than the speed of light.

Einstein also redefined **momentum** and **energy**. The new equation for momentum is:

$$p = m \cdot v = \frac{m_o \cdot v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The new equation for the **total energy** is:

$$E_{total} = m \cdot c^2$$

This can be written as:

$$E_{total} = \frac{m_o \cdot c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Einstein found that the total energy consisted of two parts.

$$E_{total} = E_{kinetic} + E_{rest\ energy}$$

The **rest energy** of a particle is given by:

$$E_{rest\ energy} = m_o \cdot c^2$$

This equation shows that matter and energy are equivalent.

The **kinetic energy** is given by:

$$E_{kinetic} = m \cdot c^2 - m_o \cdot c^2$$

This can be written as:

$$E_{kinetic} = \frac{m_o \cdot c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_o \cdot c^2$$

If v is small we have, using the binomial theorem; $1/\sqrt{(1-v^2/c^2)} = 1 + (1/2)\ v^2/c^2$.

The equation for the total energy for small v is:

$$E_{total} = \frac{m_o \cdot c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = m_o \cdot c^2 + \frac{1}{2} \cdot m_o \cdot v^2$$

The first term is the rest energy. The second term is the old term for the kinetic energy.

The Velocity Addition Law

Suppose that a spaceship moving with a velocity v relative to an observer, fires a missile with a speed u relative to the spaceship. The velocity of the missile relative to the observer is u'. It is given by the formula:

$$u' = \frac{v + u}{1 + \frac{v \cdot u}{c^2}}$$

Problems:

1) The rest mass of an electron is 9.11×10^{-31} kg. Its velocity is 0.750 c.

a) Find the mass., b) Find the momentum.

2) In a nuclear reaction, mass is converted to energy. Find the energy produced if one gram of matter is completely converted to energy. (use $E = m_o c^2$)

3)a) Use the Einstein formula ($E_k = mc^2 - m_o c^2$) to find the kinetic energy of an electron that moves with a velocity of 0.50 c.

b) Use the old ($E_k = 1/2\ m_o v^2$) formula to find the kinetic energy.

4)a) A spaceship moves relative to an observer with a velocity 0.25 c. It fires a missile forward with a velocity of 0.25 c with respect to the spaceship. Use the velocity addition formula to find the missile velocity relative to the observer.

b) Compare with the old formula for addition of velocities.

Answers: 1)a) 1.38×10^{-30} kg, b) 3.11×10^{-22} kg m/s, 2) 9.0×10^{13} J, 3)a) 1.3×10^{-14} J, b) 1.0×10^{-14}, 4)a) 0.47 c, b) b) 0.50 c.