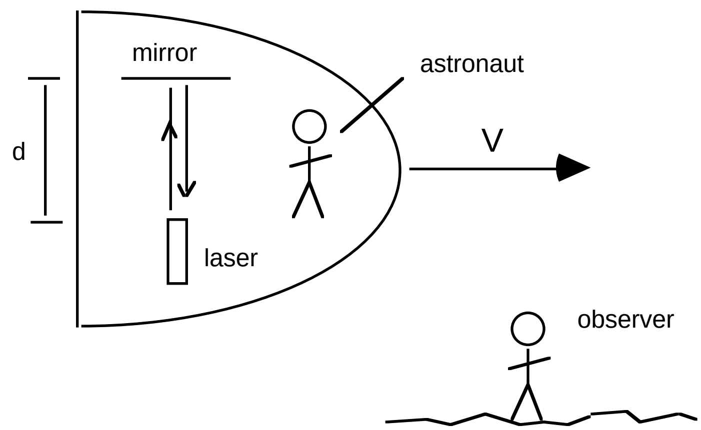
Relativity : Notes/W.S.-30

Time Dilation

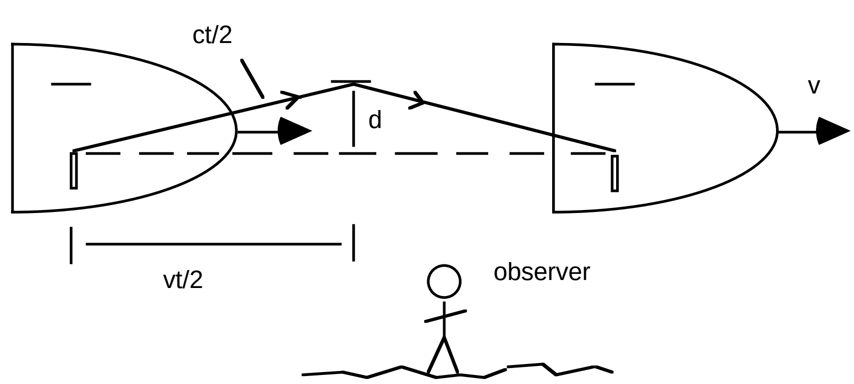
Suppose that you are an astronaut in a rocket ship. Using a laser and a mirror, you can make a light clock.



The time taken for light to travel to the mirror and back can be measured by the astronaut. It is:

$$t_o = \frac{2 \cdot d}{c}$$

The observer measures a different time for the same event!



The light ray appears to trace out part of an isosceles triangle in a time t as measured by the observer. We can find this time by using the

Pythagorean theorem. We have $[vt/2]^2 + d^2 = [ct/2]^2$. So it follows that:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is the **<u>time-dilation</u>** formula. It shows that the time an observer measures for an event on the spaceship, depends on the speed of the spaceship relative to the observer. The time t is larger than the time t_o. This phenomenon is called time dilation. It is important to note that the times measured in both instances are made using identical clocks!

Problems:

1) What does the word dilation mean?

2) Prove the time dilation formula.

3) The phenomenon of time dilation depends on the velocity of the spaceship relative to the observer. Find the time t (to 3 decimals) for an event on the spaceship as measured by an observer, if the time t_o for the event is 1.00 seconds.

V (spaceship)	t (observer)
0.010 c	_____
0.100 c	_____
0.500 c	_____
0.900 c	_____
0.990 c	_____

4) If the astronaut measures a time of 8.0 seconds for an event, find the time measured by an observer of the event, if the relative speed of the observer is:

$$\sqrt{3}/2 \cdot c$$

5) If an astronaut measures a time of 1.00 hours for an event, and the observer measures a time of 16.0 hours for the same event, what is their relative speed?

6) An observer measures a time for an event of 2.0 min. The velocity of a passing spaceship is 0.95 c. What time does an astronaut on the ship measure for the event?

Answers: 1) expansion, 2) $(vt/2)^2 + d^2 = (ct/2)^2$; $(vt/2)^2 + (ct_0/2)^2 = (ct/2)^2$; $(ct_0)^2 = (ct)^2 - (vt)^2$; $t_0{}^2 = t^2(1-v^2/c^2)$; $t = t_0 /\sqrt{(1-v^2/c^2)}$, 3) 1.000, 1.005, 1.155, 2.294, 7.089, 4) 16.0 s, 5) 0.998 c, 6) 6.4 min.