Double-Angle Identities 70

Using the sum identities, we can derive the double angle identities.

1) $\sin(2x) = \sin(x + x) = 2\sin x \cdot \cos x$

2) $\cos(2x) = \cos(x + x) = \cos^2 x - \sin^2 x$

Problems:

1)a) Show that $\cos 2x = 2\cos^2 x - 1$.

b) Show that $\cos 2x = 1 - 2\sin^2 x$

2) Show that $\tan 2x = 2\tan x/(1 - \tan^2 x)$. Use equations 1 and 2.

3) Simplify the following. Express as the sine, cosine or tangent of a single angle.

a) $2\sin 15\cos 15$

b) $1 - 2\sin^2 22.5$

c) $2\cos^2 15 - 1$

d) $\cos^2 25 - \sin^2 25$

e) $2\tan 20/(1 - \tan^2 20$

f) $2\sin 35\cos 35$

4) Prove the following identities:

a) $$\frac{2\sin^2 x}{\sin 2x} = \tan x$$

b) $$\frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$$

c) $\mathbf{1 + \cos 2x = 2\cos^2 x}$

Answers: 1)a) $\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$., b) $\cos 2x = \cos^2 x - \sin^2 x = (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$., 2) $\tan 2x = \sin 2x/\cos 2x = 2\sin x\cos x/(\cos^2 x - \sin^2 x) = (2\sin x\cos x/\cos^2 x)/((\cos^2 x - \sin^2 x)/\cos^2 x)) = 2\tan x/(1 - \tan^2 x)$., 3)a) sin30, b) cos45, c) cos30, d) cos50, e) tan40, f) sin70.