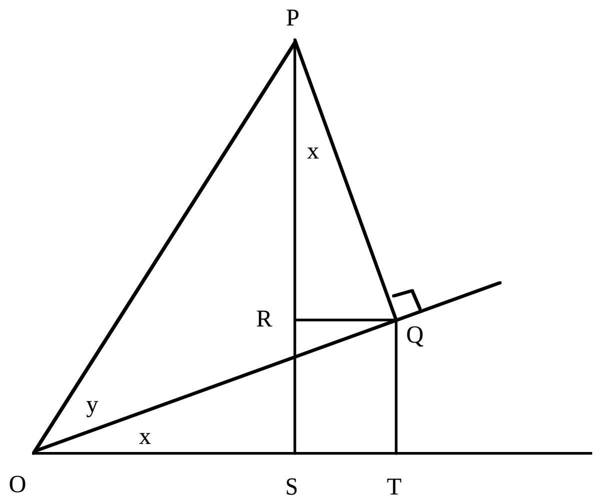
Sum and Difference Identities 60

We can derive the formula for the addition identity;

sin(x + y) = sinx cosy + cosx siny.



From the diagram above, we have;

sin(x + y) = SP/OP = SR/OP + RP/OP = TQ/OP + RP/OP

Multiply the first term by OQ/OQ and the second by QP/QP. We have;

sin(x + y) = TQ/OP OQ/OQ + RP/OP QP/QP =

TQ/OQ OQ/OP + RP/QP QP/OP = sinx cosy + cosx siny.

In a similar way, we can prove other sum and difference identities. These are given below.

Sum and Difference Identities

1) **sin (x + y) = sinx · cosy + cosx · siny**

2) **sin (x - y) = sinx · cosy - cosx · siny**

3) $\cos(x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y$

4) $\cos(x - y) = \cos x \cdot \cos y + \sin x \cdot \sin y$

5) $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$

6) $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$

Example 1:

We can use the above identities to prove many other identities.

e.g. $\cos(90° - y) = \cos(90°)\cos(y) + \sin(90°)\sin(y) = \sin(y)$.

Example 2:

We can also use these identities to find the exact values of certain angles.

e.g. Find the exact value of $\sin(75°)$.

Using the first identity above, we have; $\sin(75°) = \sin(30° + 45°) =$

$\sin(30°)\cos(45°) + \cos(30°)\sin(45°) = (1/2)(\sqrt{2}/2) + (\sqrt{3}/2)(\sqrt{2}/2) =$

$(\sqrt{2} + \sqrt{6})/4$.

Problems:

1) Express the following, in terms of the sine, cosine, or tangent of a single angle.

a) $\cos(45)\cos(30) - \sin(45)\sin(30)$

b) $\sin(60)\cos(45) - \cos(60)\sin(45)$

c) sin(15)cos(45) + cos(15)sin(45)

d) (tan(60) - tan(15))/(1 + tan(60)tan(15))

2) Simplify the following.

a) sin(90 + y)   b) cos(90 + y)   c) cos(180 - y)

3) Find the exact value.

a) sin(15)   b) cos(105)   c) tan(75)

4) Prove the formula for tan(x + y) using the formulas for sin(x + y) and cos(x + y).

Answers: 1)a) cos(75), b) sin(15), c) sin(60), d) tan(45), 2)a) cosy, b) -siny, c) -cosy, 3)a) $(\sqrt{6} - \sqrt{2})/4$, b) $(\sqrt{2} - \sqrt{6})/4$,
c) $(\sqrt{3} + 1)/(\sqrt{3} - 1) = (\sqrt{3} + 1)^2/2 = 2 + \sqrt{3}$, 4) tan(x + y) =
sin(x + y)/cos(x + y) = (sinxcosy + cosxsiny)/(cosxcosy-sinxsiny),
divide the numerator and denominator by cosxcosy. We then have
tan(x + y) = (tanx + tany)/(1 - tanxtany)