Proving Trigonometric Identities 50

It is useful to prove that a trigonometric identity is true.

There are usually several ways to prove an identity. The usual approach is to use the basic identities and the Pythagorean identities and work the left or right side down until it equals the other side as shown below.

Example 1:

Prove that; $\tan x(\tan x + \cot x) = \sec^2 x$

Left Side = $\tan^2 x + \tan x \cot x$

Left Side = $\tan^2 x + \tan x(1/\tan x)$

Left Side = $\tan^2 x + 1$

Left Side = $\sec^2 x$

Left Side = Right Side

Example 2:

Prove that; $(\tan x - 1)/(1 - \cot x) = \tan x$

Left Side = $(\sin x/\cos x - 1)/(1 - \cos x/\sin x)$

Left Side = $(\sin x/\cos x - \cos x/\cos x)/(\sin x/\sin x - \cos x/\sin x)$

Left Side = $(\sin x - \cos x)/\cos x/(\sin x - \cos x)/\sin x$

Left Side = $\sin x/\cos x = \tan x$

Left Side = Right Side

Problems:

1) Prove the following identities.

a) $\sin\theta \cdot \csc\theta = 1$

b) $\cot\theta \cdot \sin\theta = \cos\theta$

c) $\cot\theta / \cos\theta = 1 / \sin\theta$

d) $\cos\theta \cdot \csc\theta = \cot\theta$

e) $\csc\theta \cdot (1 - \sin\theta) = \csc\theta - 1$

f) $\sin\theta + \cos\theta \cdot \tan\theta = 2\sin\theta$

g) $\sin\theta (\cot\theta + \csc\theta) = \cos\theta + 1$

2) Prove the following identities.

a) $\dfrac{\csc\theta}{\sec\theta} = \cot\theta$

b) $1 + \cot^2\theta = \csc^2\theta$

c) $\dfrac{\sin\theta}{\cos\theta \cdot \tan\theta} = 1$

d) $\dfrac{1}{1 - \cos^2\theta} = \csc^2\theta$

e) $\dfrac{1 + \cot\theta}{\cos\theta} = \sec\theta + \csc\theta$

f) $\dfrac{\cos^2\theta}{1 - \sin\theta} = 1 + \sin\theta$

g) $\sec\theta - \tan\theta \cdot \sin\theta = \cos\theta$

h) $\cos^2\theta \cdot (\sec^2\theta - 1) = \sin^2\theta$

i) $\sec\theta + \tan^2\theta \cdot \sec\theta = \sec^3\theta$

j) $\tan\theta + \cot\theta = 1 / (\sin\theta \cdot \cos\theta)$

k) $(1 + \sec\theta) / \csc\theta = \sin\theta + \tan\theta$