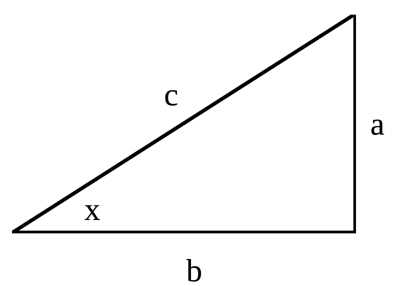
Trigonometric Identities 40

Besides the basic identities, there are also the three **Pythagorean Identities** . The first of these is;

$\sin^2 x + \cos^2 x = 1$, or $(\sin x)^2 + (\cos x)^2 = 1$.

This equation is easily proven using the Pythagorean theorem.



We have; $\sin x = a/c$ and $\cos x = b/c$, so;

$\sin^2 x + \cos^2 x = a^2/c^2 + b^2/c^2 = (a^2 + b^2)/c^2 = c^2/c^2 = 1$

The three **Pythagorean Identities** are;

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

The second and third Pythagorean identities can be proven in a manner similar to the proof given for the first Pythagorean identity.

Questions:

1) Use the Pythagorean Identities to simplify the following:

a) $1 + \tan^2\theta =$

b) $1 - \cos^2\theta =$

c) $\csc^2\theta - \cot^2\theta =$

d) $(\sec\theta - 1) \cdot (\sec\theta + 1) =$

e) $\cos\theta \cdot \sin^2\theta + \cos^3\theta =$

f) $(1 + \tan^2\theta) \cdot (\cos^2\theta) =$

g) $(\csc^2\theta - 1) \cdot (\sin^2\theta) =$

h) $\cos^2\theta - \sin^2\theta =$

i) $(\sec^2\theta - 1)/\sin^2\theta =$

j) $\cos^2\theta / (1 - \sin\theta) =$

k) $\cot\theta \cdot (\tan\theta + \cot\theta) =$

l) $\cos\theta + \tan\theta \cdot \sin\theta =$

m) $\sec\theta - \sin\theta \cdot \tan\theta =$

n) $(\sin\theta + \cos\theta)^2 - (\sin\theta - \cos\theta)^2 =$

Answers: 1)a) $\sec^2\theta$, b) $\sin^2\theta$, c) 1, d) $\tan^2\theta$, e) $\cos\theta$, f) 1, g) $\cos^2\theta$, h) $1 - 2\sin^2\theta$, i) $\sec^2\theta$, j) $1 + \sin\theta$, k) $\csc^2\theta$, l) $\sec\theta$, m) $\cos\theta$, n) $4\sin\theta\cos\theta$.