Confidence Intervals 85

We found that the true mean μ, for a population has a 68% probability of lying within σ_m of the estimate of the mean which is $\bar{x}$. So we have the inequality;

$$\bar{x} - \sigma_m < \mu < \bar{x} + \sigma_m$$

A similar problem involves estimating the true probability **p**, (or proportion or percentage) for an event from a sample of the total population.

We can derive an inequality similar to the one above.

$$\bar{p} - \sigma_p < p < \bar{p} + \sigma_p$$

or;

$$\bar{p} - \frac{\sigma}{n} < p < \bar{p} + \frac{\sigma}{n}$$

Here, n is the sample size, and the estimated probability is $\bar{p}$. We assume a binomial distribution, so the standard deviation for the sample is $\sigma = \sqrt{npq}$. We have;

$$\bar{p} - \sqrt{\frac{\bar{p}\,\bar{q}}{n}} < p < \bar{p} + \sqrt{\frac{\bar{p}\,\bar{q}}{n}}$$

This inequality says that there is a 68% chance that the true probability p lies within $\sqrt{\frac{\bar{p}\,\bar{q}}{n}}$ of the estimated probability $\bar{p}$. For other confidence intervals we can write;

$$\bar{p} - Z\sqrt{\frac{\bar{p}\,\bar{q}}{n}} < p < \bar{p} + Z\sqrt{\frac{\bar{p}\,\bar{q}}{n}}$$

For a 68% confidence interval, Z = 1. For a 95% confidence interval, Z = 1.96 or about 2 and so on.

The quantity $\sqrt{\frac{\bar{p}\,\bar{q}}{n}}$ is called the **Standard Error**. The quantity $Z\sqrt{\frac{\bar{p}\,\bar{q}}{n}}$ is called the **Margin of Error**.

Questions:

1) It is found that the probability for an event is 0.25 when the sample size is 500. Find the 68% confidence interval.

2) A small college has a student population of 8500. It is known that 28% of the students own a car.

a) Find the standard error.

b) Find the 68% confidence interval for the percentage of **all** college students who own a car.

c) Find the 95% confidence interval.

3) Use the table to calculate Z for a confidence interval of 99%.

Answers: 1) $0.23 < p < 0.27$, 2)a) 0.0049, b) $27.5 < p < 28.5$, c) $27 < p < 29$, 3) 2.58.