Confidence Intervals 80

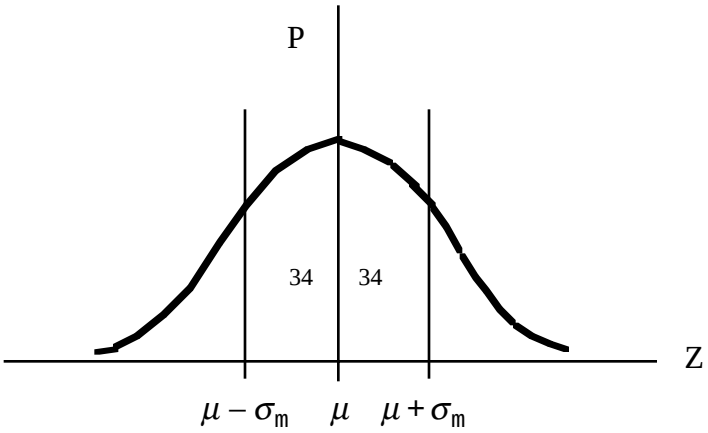
The **true** mean for a very large set of data is usually given by the symbol μ (mu).

We can only estimate μ because our sample is restricted to a small part of the total data set.

For example, if μ is really 23.3, we may find sample means ($\overline{\mathbf{X}}$) of 23.7 or 22.9 and so on.

We assume that the sample means are distributed normally about the true mean μ, with a standard deviation of sigma = σ_m .

Distribution of Sample means about True Mean



It turns out that the standard deviation σ_m , of the set of sample means $\{\overline{x}_1, \overline{x}_2, \overline{x}_3 ...\} = \sigma / \sqrt{n}$, where n is the number of data points in a sample and sigma σ is the standard deviation for that sample.

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

Example: Suppose that we have sample data on the mass of a bolt in grams. The measurements are; 6.12, 6.14, 6.10, 6.12, 6.13, 6.12, 6.11, and 6.13.

The mean $\bar{x}$, = 6.121, and σ = 0.012. Since n = 8, we have σ_m = 0.004. So;

$$\bar{x} - \sigma_m < \mu < \bar{x} + \sigma_m$$

This means that there is a 68% chance that μ is between 6.121 ± 0.004. We say that the confidence interval is 68%.

We can have a confidence interval of 95% by using a z-score of 1.96 (or about 2). So;

$$\bar{x} - 2\sigma_m < \mu < \bar{x} + 2\sigma_m$$

There is a 95% chance that the true mean μ, is between 6.121 ± 0.008.

Problems:

1) A sample has N = 64 with $\bar{x}$ = 85 and σ = 12.

a) Find the 68% confidence interval.

b) Find the 95% confidence interval.

2) The mean mass for 17 year old girls in a class is found to be 43 kg. The sample size is a class of 25 girls. The standard deviation is 3.5 kg. Find the 95% confidence interval for the true mean mass of 17 year old girls. The class is assumed to be a random sample of the population of 17 year old girls.

3)a) Find the 90% confidence interval for the example.

b) Find the 80% confidence interval for the example.

Answers: 1)a) 83.5 < μ < 86.5, b) 82 < μ < 88, 2) 41.6 < μ < 44.4, 3)a) z = 1.65, 6.114 < μ < 6.128, b) z = 1.28, 6.116 < μ < 6.126,