The Normal Approximation to the Binomial Distribution 70

The binomial distribution can be approximated by a normal distribution. This can make solving certain problems much simpler.

The approximation works well if Nxp > 5 and Nxq > 5.

Example 1: Suppose that 16 coins are tossed. We might want to know the probability that less than 7 heads are tossed. The probabilities (to six decimals) are calculated according to the binomial distribution formula.

P(0) = 0.000015, P(1) = 0.000244, P(2) = 0.001831, P(3) = 0.008545, P(4) = 0.027771, P(5) = 0.066650, P(6) = 0.122192, P(7) = 0.174561, P(8) = 0.196381, P(9) = 0.174561, P(10) = 0.122192, P(11) = 0.066650, P(12) = 0.027771, P(13) = 0.008545, P(14) = 0.001831, P(15) = 0.000244, P(16) = 0.000015.

The probability that less than seven heads are tossed is equal to the sum of the probabilities P(0) to P(6) = 0.2272 (to four decimals).

Example 2: We can also solve this problem by assuming that the binomial distribution is “normal”. This makes the problem much simpler.

Since N = 16, and p = 0.5, then Nxp = Nxq = 8 > 5. The approximation should work.

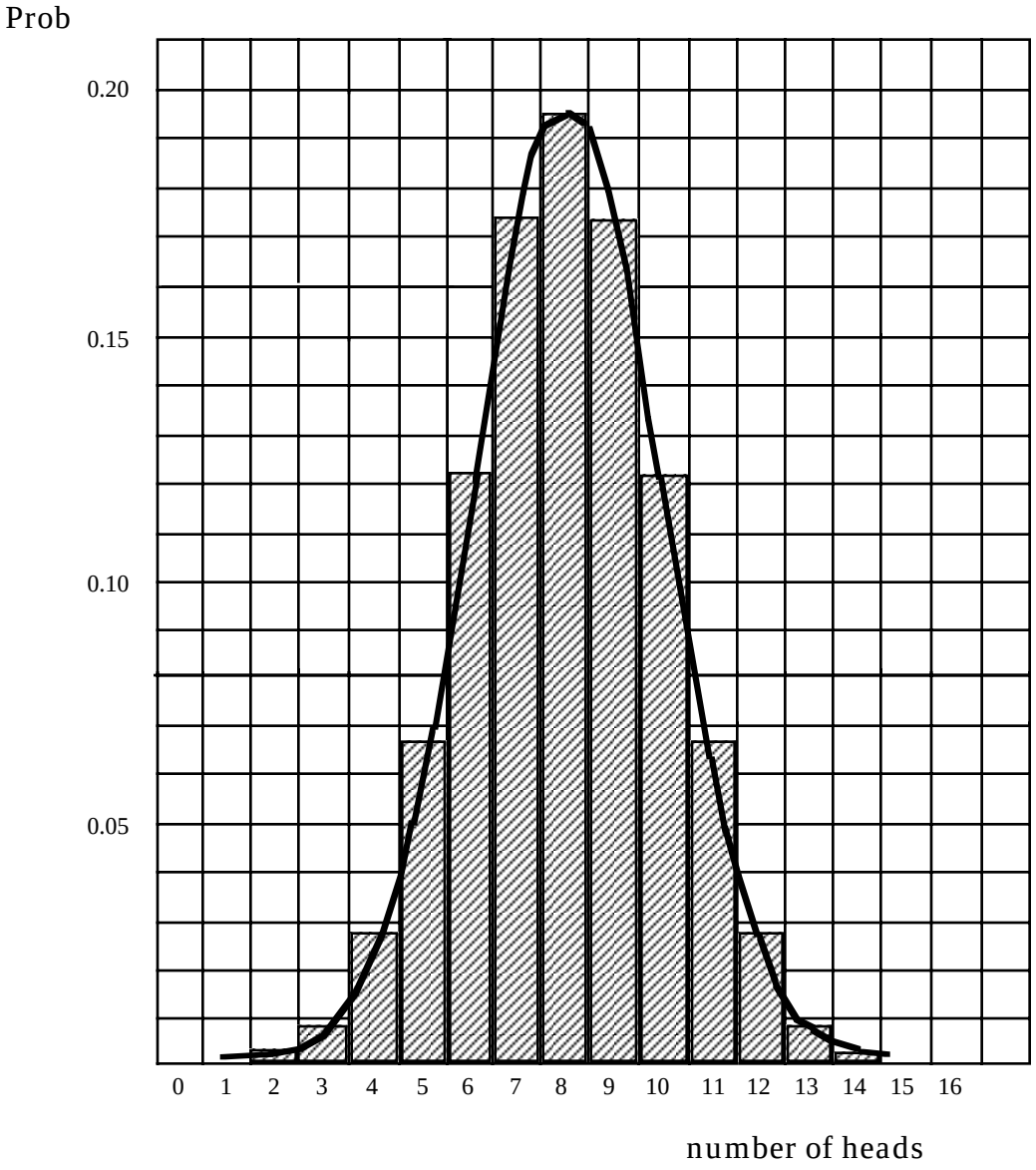
The mean = Nxp = 8. The standard deviation = $\sqrt{Nxpxq}$ = 2.0

We can calculate the z-score. It is (6.5 - 8)/2 = -0.75. We use 6.5 as the binomial distribution is discrete and the normal distribution is continuous.

The corresponding probability from the table is 0.2266.

The normal approximation works well in this case. It gets better when N is larger.

Note: The binomial distribution < 7, is approximated by a normal distribution < 6.5. See the diagram below.



Example 3: Suppose that we want to know the probability of tossing 5 to 10 heads inclusive. It is P(5) + P(6) + P(7) + P(8) + P(9) + P(10) = 0.8565 to four decimals.

We can also calculate the probability by using the normal distribution approximation. The z-scores are (4.5-8.0)/2 = -1.75, and (10.5-8.0)/2 = 1.25. The corresponding area (probability) = 0.8543.

Problems:

1) Suppose that 20 coins are tossed. Use the normal distribution approximation to calculate the following probabilities to 2 decimals.

a) P(more than 12 heads) b) P(6-9 heads inclusive)

2) The probability that a factory produces a defective light bulb is 0.012. A shipment contains 20,000 bulbs.

a) Find the mean number of defective bulbs in a shipment.

b) Find the standard deviation.

c) Find the probability that the shipment contains more than 250 defective bulbs.

3) A die is rolled 300 times.

a) What is the mean number of fives rolled?

b) What is the standard deviation?

c) Assume a normal distribution for the number of fives rolled. Find the probability of rolling < 40 fives (to three decimals).

Answers: 1)a) 0.13, b) 0.39, 2)a) 240, b) 15.4, c) 0.25, 3)a) 50, b) 6.5, c) 0.052.