The Standard Normal Distribution 60

We need to convert "raw data" to z-scores in order to compare two sets of data with different means and standard deviations.

If this is done and the data is distributed normally, the data will fit what is called the **Standard Normal Distribution** . All normal distributions can be converted to this type of distribution.

The Standard Normal Distribution has these properties:

- the mean is zero.
- the standard deviation is one.
- the area under the curve is one.
- 68% of the data are between -1 and 1.
- 95% of the data are between -2 and 2.
- 99.7% of the data are between -3 and 3.

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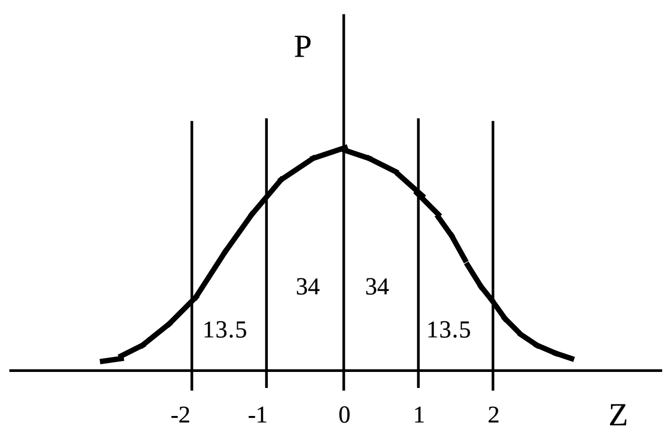

The Standard Normal Distribution Table gives the probability of a measurement being less than a particular z-score. We can use it to find the probability of a measurement being between two particular z-scores.

example 1:

$P(z < -1) = 0.1587$

example 2:

$P(-0.5 < z < 1.2) = P(z < 1.2) - P(z < -0.5)$

$= 0.8849 - 0.3085 = 0.5764$

example 3:

A large group of students wrote an exam. The marks were distributed normally. The mean was 110 and the standard deviation was 15.

Find the percentage of students who had a mark between 95 and 125.

Answer: It is 68% because these marks are one standard deviation from the mean.

Find the percentage of students who had a mark between 90 and 120.

We calculate the z-scores. They are -1.33 and 0.67 respectively. These correspond to areas 0.0918 and 0.7486 from the table. The difference is 0.6568 = 65.7%

Problems:

1) Give four important properties of the Standard Normal Distribution.

2) Find the probabilities (area under curve).

a) $P(z < 0.6)$ b) $P(z < -1.3)$ c) $P(-1 < Z < 1)$ d) $P(0.4 < z < 1.5)$

3) The following are areas under the curve. Find the corresponding z-score.

a) P = 0.85   z < _____   b) P = 0.40   z < _____

4) A set of exam marks, x_i, are distributed normally with a mean of 120 and a standard deviation of 16. Find the following probabilities. Convert to z-scores first.

a) $P(x < 136)$ b) $P(x < 96)$ c) $P(104 < x < 136)$

d) What was the minimum mark for students in the top 10%?

5) A shipment of oranges has a mean diameter of 9.0 cm with a standard deviation of 1.4 cm. The diameters are normally distributed. What percentage of the oranges have a diameter between 8.0 cm and 10.0 cm?

6) A police force in a small town has uniforms to fit men with heights between 173 cm and 188 cm. Assume that men's heights are distributed normally with a mean of 178 cm and a standard deviation of 6.4 cm. What percentage of men may be fitted?

Answers: 1) mean = 0, S.D. = 1, area under curve = 1, 68% of data (z-scores) is between -1 and 1., 2)a) 0.7257, b) 0.0968, c) 0.6826, d) 0.2778, 3)a) 1.04, b) -0.25, 4)a) 0.8413, b) 0.0668, c) 0.6826, d) 140, 5) 52%, 6) 72%