Z-Scores 50

It is often convenient to transform a set of data x_i into a new set z_i. If the mean and standard deviation of the set (x_i) are known, then we can calculate the z-scores (z_i) for each of the data points in the set x_i.

The formula for the z-score is given below.

$$z_i = \frac{x_i - \bar{x}}{\sigma}$$

Converting to z-scores makes it easier to compare two sets of data with different means and standard deviations.

Example:

An 11 year girl has a weight of 42 kg. The mean and standard deviation for 11 year old girls are 38 kg and 5 kg respectively. Her 14 year old sister has a weight of 53 kg. The mean and standard deviation for 14 year old girls are 49 kg and 6 kg.

Which girl is heaviest for her age?

We calculate the z-scores. For the 11 year old it is (42-38)/5 = 0.80. For the 14 year old it is (53-49)/6 = 0.66.

The z-score for the 11 year old is bigger, therefore, she is heavier for her age. Her weight is more standard deviations from the mean.

Problems:

1) Find the z-scores (to one decimal) for two 17 year old boys. John's height is 168 cm. Bob's height is 182 cm. For 17 year old boys, the mean height is 177 cm and the standard deviation is 6 cm.

John _____ Bob _____

2) Find the z-scores for the following set of quiz marks.

x_i	4	6	7	7	8	8	9

z_i __ __ __ __ __ __ __

3) Bob wrote entrance exams for college A and college B.

At college A he scored 76%. The mean was 72 and the standard deviation was 10. Find his z-score.

At college B he scored 80%. The mean was 77 and the standard deviation was 12. Find his z-score.

On which exam did he score the better mark? Explain.

Answers: 1) -1.5, +0.8, 2) mean = 7.0, S.D. = 1.5, z-scores are respectively, -2.0, -0.7, 0.0, 0.0, 0.7, 0.7, 1.3, visit www.mrowen.com, 3) 0.40, 0.25, The college A score is better as 0.4 > 0.25. It is more standard deviations from the mean.