Statistics Mean and Standard Deviation 10

When dealing with a large number of measurements or observations, it is useful to find the average, or mean of the set. In this way, one number can be used to represent the set. The **mean** is the sum of the measurements in the set divided by the number of measurements in that set.

The mean (x-bar) of the set { x_1, x_2, x_3, ... x_N } is given by the formula:

$$\bar{x} = \{ x_1 + x_2 + . . + x_N \} / N$$

Example 1: The mean of the set of numbers {1.419, 1.428, 1.441, 1.450, 1.451, 1.459, 1.460, 1.460, 1.469, 1.470, 1.477, 1.500} is 1.457.

The mean is useful, but generally we need another number to represent the set because two sets may have the same mean, but are spread about the mean in a very different manner. The spread is given by the **standard deviation**.

The standard deviation (sigma) is given by the formula:

$$\sigma = \sqrt{\frac{\{ (x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_N - \bar{x})^N \}}{N}}$$

Example 2: Find the standard deviation of the set of numbers given in the previous example.

Using the above formula we find that the standard deviation is 0.021 to within three decimal places.

The data set in example 1 can be represented by 1.457 ± 0.021. The majority of the data are within one standard deviation of the mean. If the standard deviation is small for a set of data, then most of the measurements are close to the mean.

Problems:

1) Verify that the mean and standard deviation given in the examples are correct.

2) A teacher gives a quiz to fifteen students. The quiz is out of ten marks. The marks that the students get are {3, 4, 5, 5, 6, 6, 7, 7, 7, 7, 8, 8, 9, 10, 10}.

a) Find the average mark (to within one decimal).

b) Find the standard deviation (to within one decimal).

3) The velocity of light (in km/s) is found by experiment. Six different scientists find the following values for the speed of light.

299,792, 299,784, 299,789, 299,799, 299,787, 299,795.

a) Find the average speed in km/s.

b) Find the standard deviation in km/s.

Answers: 2)a) 6.8, b) 2.0, 3)a) 299,791 km/s, b) 5 km/s.