

Conditional Probabilities 40

Sometimes, events A and B are NOT independent. If event B occurs first, then the probability that event A occurs, is dependent on the occurrence of event B.

The probability that A will occur given that event B has already occurred, is called a conditional probability. It is given by the formula:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

This is equivalent to the formula:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Example 1:

A die is rolled. Find the probability that a prime number has been rolled, given that the number is even. (prime = 2, 3, 5). Use the first formula.

$$P(\text{prime} | \text{even}) = P(\text{prime and even})/P(\text{even})$$

$$P(\text{prime} | \text{even}) = (1/6)/(3/6) = 1/3$$

Example 2:

Suppose that two cards are drawn from a standard shuffled deck. Find the probability of drawing a black king and a red queen, if the black king has already been drawn. Use the second formula.

$$P(\text{BK and RQ}) = P(\text{BK}) \times P(\text{RQ} | \text{BK})$$

$$P(\text{BK and RQ}) = (2/52) \times (2/51) = 1/663$$

Problems:

- 1) Write down the formula for $P(C | D)$
- 2) Given that $P(A) = 1/2$, $P(B) = 1/4$, $P(A \text{ and } B) = 1/5$

Find the following:

a) $P(A | B)$

b) $P(B | A)$

3) At a particular school, the probability that a student has blue eyes is 0.15. The probability that a student has light hair and blue eyes is 0.10. Find the probability that a student has light hair given that their eye color is blue.

4) Two cards are drawn from a standard shuffled deck. Find the probability of drawing a spade, if a spade has already been drawn.

5) In a small school of 110 students, 75 students take English, 65 take math, and 30 take both subjects. Find the probability of taking English, given that a student is taking math. Find $P(E | M)$.

6) A bag contains three white and five black marbles. One marble is removed. Then a second marble is removed without replacing the first marble. Find the following:

a) $P(W \text{ and } B)$

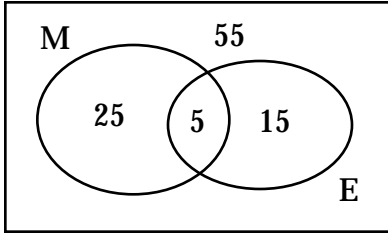
b) $P(B \text{ and } B)$

7) Find the probability of drawing a queen from a standard shuffled deck of cards given that it is a face card.

8) In a college, 30% of students fail math, 20% of students fail English and 5% fail both math and English. Find the probability that a student fails math given that they passed English. Draw a Venn diagram.

9) A die is tossed. Find the probability that the outcome is less than a 5 given that the number is odd.

Answers: 1) $P(C \text{ and } D)/P(D)$, 2)a) $4/5$, b) $2/5$, 3) $2/3$, 4) $\{13/52 \times 12/51\} / \{1/4\} = 4/17$, 5) $30/65 = 6/13$, 6)a) $15/56$, b) $5/14$, 7) $\{4/52\} / \{12/52\} = 1/3$, 8) $25/80 = 5/16$, (25% fail math and pass English),



9) $\{2/6\}/\{3/6\} = 2/3.$