Probability 35

Use the factorial, permutation and combination formulae to solve the following probability problems.

Example 1:

A briefcase has a combination lock. Four digits, (0-9), must be chosen in the correct order to open the case. What is the probability of choosing the correct order by chance?

Answer: $1/(10x10x10x10) = 0.0001$

Find the probability if the digits must all be different.

Answer: $1/{}_{10}P_4 = 1/10x9x8x7 = 0.0002$

Example 2:

Five people form a line up. What is the probability that they line up, according to age, from youngest to oldest?

Answer: $1/1x2x3x4x5 = 1/120 = 0.0083$

Example 3:

Five horses are in a race. Find the probability that a person selects the horses which finish first and second.

There is one way to choose first place, one way to choose second place, and 3x2x1 ways to choose the remaining three places.

The total number of possibilities is 5!

So the required probability is 1x1x3x2x1/5! = 1/20.

Example 4: A poker hand consists of five cards. What is the probability that they are all hearts?

The probability equals the number hands containing only hearts divided by the total number of hands = ${}_{13}C_5/{}_{52}C_5 = 0.000495$

Example 5:

A bag contains 3 black marbles and 4 white marbles. Two marbles are chosen at the same time (without replacement). Find the probabilities. (Note: this problem can also be solved by using a tree diagram)

P(2 black) = ${}_3C_2/{}_7C_2 = 1/7$

P(2 white) = ${}_4C_2/{}_7C_2 = 2/7$

P(1 white and 1 black) = $\{{}_4C_1 \times {}_3C_1\}/ {}_7C_2 = 4/7$

The sum of the above three probabilities is 1.0.

Questions:

1) A combination lock requires that three different numbers be dialed in the correct order. The numbers are from 1 to 30. Find the probability that someone could open the lock by randomly choosing a three number combination.

2) Four cards are numbered from 1 to 4 and placed in a box. What is the probability that the four cards will be drawn in order from 1 to 4?

3) Eight horses are in a race. Find the probability of selecting the horses which come in first, second and third.

4) Words are made using the letters of the word seven. Answer the following questions.

a) How many different words are possible?

b) Find the probability that both e's are together. (Hint; treat the pair of e's as if they were one "letter")

c) Find the probability that the e's are at both ends of the word.

5) Two cards are drawn from a standard deck. Find the probabilities.

a) P(both are black) b) P(both are aces)

6) Three cards are chosen from a shuffled standard deck.

a) How many three card hands are there?

b) What is the probability of getting three reds?

c) What is the probability of getting three hearts?

d) What is the probability of getting one club and two diamonds.

7) A committee of four people is selected from a group of nine people. There are six men and three women in the group. Find the probabilities.

a) The committee has four men.

b) The committee has two men and two women.

c) The committee has one man and three women.

8) A bag contains five black and seven white marbles. Two marbles are picked. Find the following probabilities.

a) P(two black) b) P(two white)

c) P(one white and one black, order not important)

d) Find the sum of the previous answers.

Answers: 1) $1/{}_{30}P_3$ = 1/30x29x28 = $4.1x10^{-5}$, 2) 1/4! = 1/24 = 0.042, 3) 1x1x1x5!/8! = $3.0x10^{-3}$, 4)a) 5!/2! = 60, b) (treat the two e's as one letter), 4!/60 = 2/5, c) (permute the remaining three letters), 1x3x2x1x1/60, = 1/10, 5)a) ${}_{26}C_2/{}_{52}C_2$ = 25/102, b) ${}_4C_2/{}_{52}C_2$ = 1/221, 6)a) ${}_{52}C_3$ = 22,100, b) ${}_{26}C_3/{}_{52}C_3$ = 2/17, c) ${}_{13}C_3/{}_{52}C_3$ = 11/850, d) ${}_{13}C_1x{}_{13}C_2/{}_{52}C_3$ = 39/850, = 0.046, 7)a) ${}_6C_4/{}_9C_4$ = 5/42, b) ${}_6C_2x{}_3C_2/{}_9C_4$ = 5/14, c) ${}_6C_1x{}_3C_3/{}_9C_4$ = 1/21, 8)a) ${}_5C_2/{}_{12}C_2$ = 5/33, b) ${}_7C_2/{}_{12}C_2$ = 7/22, c) ${}_5C_1x{}_7C_1/{}_{12}C_2$ = 35/66, d) 1.0.