Probability 20

In more complex probability problems we may want to know the probability of two events occurring.

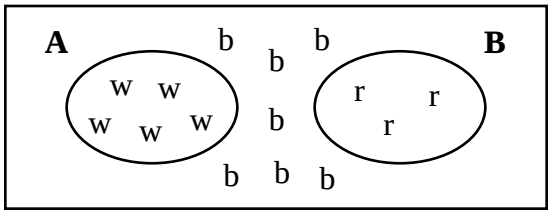
A pair of events may be **<u>mutually exclusive</u>**.

If two events A and B, are mutually exclusive, they cannot happen at the same time.

For example; event A, may represent drawing a white marble from a bag of white, red and blue marbles. Event B, may represent drawing a red marble. These events are mutually exclusive.

Suppose that we want to know the probability of drawing a white marble or a red marble. If there are five white marbles, three red marbles, and seven blue marbles in the bag, then P(W or R) = P(W) + P(R) = 5/15 + 3/15 = 8/15.

This situation may be represented by the Venn diagram below.



P(W and R) = 0. This means that both events cannot happen at the same time. The marble drawn is either white or red.

Some events are **<u>not mutually exclusive</u>**.

For example; suppose that we have twelve face cards from a standard deck of cards. We draw one card. Event A is the drawing of a red card. Event B is the drawing of a king.

What is the probability of drawing a red card or a king? In this case we can draw a red card and a king (red king) at the same time.

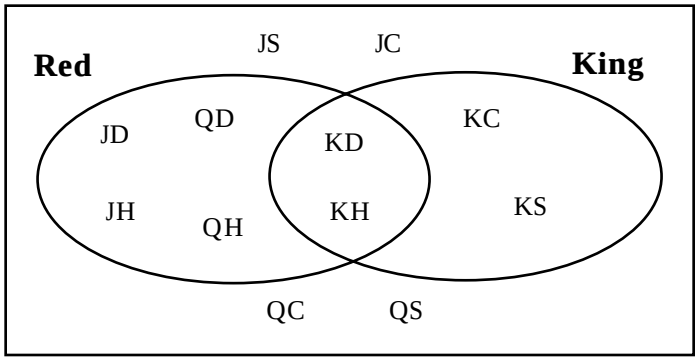
The probability of drawing a red card is 1/2. The probability of drawing a king is 1/3. The probability of drawing a red king is 1/6.

The probability of drawing a red card or a king equals 1/2 + 1/3 - 1/6 = 8/12 = 2/3.

P(Red or King) = P(Red) + P(King) - P(Red King)

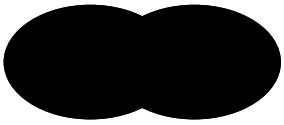
We have to subtract the probability of getting a red king because this possibility is counted <u>twice</u> in P(Red) + P(King). There is an overlap of the two events.

The Venn diagram is shown below.



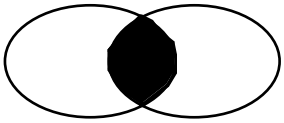
In general, for two events A and B, we have the **<u>Addition Law</u>**:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



Event A Event B

A OR B



Event A Event B

A AND B

In the above situation, the probability of getting a red card AND a king (red king) equals 1/2 x 1/3 = 1/6

In general we have the **Multiplication Law**:

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

This says that the probability of events A and B happening at the same time equals the product of the two individual probabilities. This assumes that the probabilities are independent. This means that event A does not depend on event B or B does not depend on A.