Solving Problems Using Logs 70

Often, we need to solve problems such as

$$3^x = 15$$

We could use a calculator to 'guess' the solution, but it turns out that it is much easier to use the laws of logarithms to solve the problem. Take logarithms to base 10 of both sides of the above equation.

$$\log 3^x = \log 15$$
$$x \log 3 = \log 15$$
$$x = \log 15/\log 3$$
$$x = 1.176/0.477 = 2.465$$

We can use the laws of logarithms to change one base to another base. For example:

$$\log_2 7 = x$$
$$2^x = 7$$
$$x \log 2 = \log 7$$
$$x = \log 7/\log 2$$
$$\log_2 7 = \log 7/\log 2 = 2.8$$

In general, we have;

$$\log_a c = \frac{\log_b c}{\log_b a}$$

Often, we use b = 10, so we can calculate $\log_a c$ using a calculator.

Example: Solve for x.

$5^x = 16^{2x - 4}$ -> $x \log 5 = (2x - 4) \log 16$ ->

$0.699\ x = 1.2\ (2x - 4)$ -> $x = 2.82$

Example: Solve for the time t.

The population of a country increases at a rate of 2.5% per year. Find the time required for the population to double.

The equation to be solved is: $2 = 1x(1.025)^t$

Taking logs of both sides: $\log 2 = t \times \log (1.025)$

solving, we have $t = (\log 2)/(\log 1.025) = 28.1$ yr.

Problems:

1) Solve for x.

a) $7^x = 22.5$ b) $12^x = 0.00013$

2) Solve for x.

a) $5^x = 2^{x-1}$ b) $8^{2x} = 10^{x+2}$

3) Use the formula to evaluate.

a) $\log_5 0.75$ b) $\log_6 360$

4) A town has a population of 120,000. If the population grows at a rate of 5.0% per year, find the time when the population will be 180,000.

5) Suppose that $5000 is deposited in a bank. It earns 8.5% interest per year compounded annually. In how many years will it take for the for it to grow to $12,000?

6) A new car is worth $22,000. The rate of depreciation is 14%. When will the car be worth $14,000?

Answers: 1)a) 1.6, b) -3.6, 2)a) -0.75, b) 2.5, 3)a) -0.179, b) 3.285, 4) $180{,}000 = 120{,}000x(1.05)^t$; t = 8.31 years, 5) $12{,}000 = 5000x(1.085)^t$; t = 10.73 years, 6) $14{,}000 = 22{,}000x(0.86)^t$; t = 3.00 years.