Laws of Logarithms 60

When dealing with exponents, there are rules.

Let A = 10^2, and let B = 10^3.

We have: A x B = 10^2 x 10^3 = 10^{2+3} = 10^5. We add the exponents.

Using logarithms, we have:

log A + log B = 2 + 3 = 5 = log 10^5 = log AxB, or,

log AB = log A + log B.

It can be shown that this rule works for any base.

Using similar reasoning, we can find other laws for logarithms.

The basic laws for logarithms are given below. The base can be any positive number.

$$\mathbf{Log\ (X \cdot Y) = Log\ (X) + Log\ (Y)}$$

$$\mathbf{Log\ (X / Y) = Log\ (X) - Log\ (Y)}$$

$$\mathbf{Log\ (x^n) = n \cdot Log\ (x)}$$

Questions:

1) Use the laws to simplify and evaluate:

a) log500 + log200 b) $\log_2 8 + \log_2 32$ c) $\log_5 625 - \log_5 125$

d) $\log_4 32 + \log_4 2$ e) $\log_2 320 - \log_2 40$ f) $\log_9 27 + \log_9 243$

2) Express as a sum or difference of logarithms.

a) log a^2 b) log (a b)

c) log (a/c) d) log (a b/c)

3) Express as a single logarithm.

a) log u + log v b) log u + log u

c) log u - log v d) log u + log v + log w

e) log u - log v + log w f) 2log u + 3logv

4) Solve for x.

a) log 2 + log x = log 10 b) log x + log x = log 16

c) log 6 - log x = log 2 d) log 7 + log (x+2) = log 56

5) Let $\log_2 5 = a$, and let $\log_2 3 = b$. Write the following in terms of a and b.

a) $\log_2 25$ b) $\log_2 15$ c) $\log_2 20$

d) $\log_2 75$ e) $(\log_2 5)/(\log_2 3)$ f) $\log_2(5/9)$

Answers: 1)a) 5, b) 8, c) 1, d) 3, e) 3, f) 4, 2)a) log a + log a, b) log a + log b, c) log a - log c, d) log a + log b - log c, 3)a) log uv, b) log u^2, c) log (u/v), d) log (uvw), e) log (uw/v), f) log(u^2v^3), 4)a) 5, b) 4, c) 3, d) 6, 5)a) 2a, b) a + b, c) 2 + a, d) 2a + b, e) a/b, f) a - 2b.