

Infinite Geometric Series 50

Some series are infinite. An infinite geometric series may or may not have a sum.

If r is ≥ 1 , or ≤ -1 , the sum is infinite.

If $-1 < r < 1$, the sum is finite.

Using the series formula, we find for $-1 < r < 1$, since $r^\infty = 0$,

$$S_\infty = \frac{a(r^\infty - 1)}{r - 1} = \frac{a(-1)}{r - 1} = \frac{a}{1 - r}$$

Example:

Suppose that we have the infinite series with $a = 4$, and $r = 1/2$.

Using the above formula, the series; $4 + 2 + 1 + 1/2 + \dots = 4/(1-1/2) = 8$.

Problems:

1) Find the sum of each infinite geometric series if there is one.

a) $a = 9$, $r = 1/3$

b) $a = 1$, $r = 1/10$

c) $a = -5$, $r = -1/2$

d) $a = 1/8$, $r = 2$

e) $a = 65$, $r = -1/4$

f) $a = 3$, $r = 1/\sqrt{3}$

2) Write the following numbers as an infinite series. Use the formula to find the sum of the series to write the sum as a rational number.

a) $0.5555\dots$

b) $0.37373737\dots$

3) An infinite geometric series has a first term of 100 and a sum of 90. Find r .

4) An oil well produces 100,000 barrels of oil per year. The amount produced decreases by 2% per year. Find the total lifetime production of the well. (use an infinite series to approximate this situation and find the sum)

5) A ball is dropped from a height of 64 cm. On each bounce, it reaches one-half of the previous height. Estimate the total distance traveled.

Answers: 1)a) $27/2$, b) $10/9$, c) $-10/3$, d) not possible, e) 52, f) 7.1,
2)a) $5/9$, b) $37/99$, 3) $-1/9$, 4) 5,000,000 barrels, 5) 192 cm.