

Geometric Series 40

The sum of n terms of a geometric series is S_n .

$$S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

If we multiply by r we have;

$$r S_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$$

Subtracting the first equation from the second, we have;

$$(r - 1) S_n = ar^n - a$$

or,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Example:

Use the formula to find the sum of the first six terms of the series:

$$4 + 12 + 36 + 108 + 324 + 972$$

We have; $a = 4$, $r = 3$, $n = 6$,

$$S_6 = \frac{4(3^6 - 1)}{3 - 1}$$

The sum $S_6 = 1456$.

Problems:

1) Use the formula to find the sums. Write down each series and add each term directly to verify the formula.

a) $a = 16, r = 1/2, n = 5$

b) $a = 5, r = 2, n = 6$

c) $a = 27, r = -1/3, n = 4$

d) $a = 4, r = 3, n = 8$

2) Use the formula to find the sum.

$$3^0 + 3^1 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 + 3^7 + 3^8 + 3^9 + 3^{10}$$

3) Use the formula to find the first term of a series, given that $S_7 = 344$ and $r = -2$.

4) How many ancestors do you have going back six generations?

5) Use the formula to find the sum.

$$S = 5 + 20 + 80 + \dots + 81,920$$

Answers: 1)a) 31, b) 315, c) 20, d) 13,120, 2) 88,573, 3) 8, 4) 126, 5) 109,225.