

Geometric Sequences 20

Suppose that we have the geometric sequence:

$$8, \quad 16, \quad 32, \quad 64, \quad \dots$$

We say that the geometric mean of 8 and 32 is 16. The geometric mean of 16 and 64 is 32. The two geometric means between 8 and 64 are 16 and 32.

Example:

Find the geometric mean between 2 and 32. We have $a = 2$, and $ar^2 = 32$. Solving for r , we find $r = \pm 4$. The geometric mean is ar , the term in the middle. This gives a mean of ± 8 .

Similarly, two geometric means can be found between two numbers. Let the first number be a and let the second number be ar^3 . Solve for r . The two means are; ar , and ar^2 .

Problems:

1) Which of the following sequences are geometric? (state yes or no)

a) 1, 4, 16, 64, ...

b) 9, 16, 25, 36, ...

c) 3, 5, 9, 17, ...

d) -2, $\frac{2}{3}$, $-\frac{2}{9}$, $\frac{2}{27}$, ...

2) Find one geometric mean between the following pairs of numbers.

a) 3 and 48

b) $\frac{1}{5}$ and 45

c) 1 and 5

d) -4 and -1

3) Find two geometric means between the following pairs of numbers.

a) 8 and 216

b) 32 and -4

c) 3 and 375

d) $\frac{2}{3}$ and 144

4) Find the missing terms in the following geometric sequences.

a) 1 , ____, 4 , ____

b) ____, ____, 32 , 256

c) 40 , ____, ____, 5

d) 2 , ____, 24 , ____

Answers: 1)a) yes, b) no, c) no, d) yes, 2)a) ± 12 , b) ± 3 , c) $\pm\sqrt{5}$, d) ± 2 ,
3)a) 24, 72, b) -16, 8, c) 15, 75, d) 4, 24, 4)a) ± 2 , ± 8 , b) $1/2$, 4, c)
20,10, d) $\pm 4\sqrt{3}$, $\pm 48\sqrt{3}$.