Geometric Sequences 10

A geometric sequence is one in which a term is formed by multiplying the previous term by some number.

Example of a geometric sequence: 3,   12,   48,   192 ....

In this sequence, the first term is 3, and each successive term is found by multiplying the previous term by 4.

All geometric sequences have the following form.

$$t_1, \ t_2, \ t_3, \ t_4, \ \ldots$$

The terms are t_1, t_2, t_3, and so on. The nth term in the sequence is given by t_n.

$$t_n = a \cdot r^{\,n-1}$$

We can write the sequence as;

$$a, ar, ar^2, ar^3, \ldots$$

The **first term** of the sequence is a. The number that multiplies the previous term is r. The number r is called the **common ratio**. The first term and the common ratio can be any real number.

Problems:

1) Give the next three terms in each sequence.

a) 1/3, 1, 3, ...   b) 1024, 128, 16, ...

c) √2, 2, 2√2, ...   d) -96, 48, -24, ...

2) Write down the first three terms.

a) a = 5, r = 2   b) a = 8, r = 1/2

c) $a = 512$, $r = -1/4$ d) $a = -1/27$, $r = 9$

3) Write down the first three terms. ($n = 1, 2, 3$)

a) $t_n = 2^{n-1}$ b) $t_n = 25(1/5)^{n-1}$

c) $t_n = (-3)^n$ d) $t_n = 2(4)^{n-2}$

4) A geometric sequence is given by:

$$-729, \ 243, \ -81, \ 27, \ ...$$

a) What is the value of the first term a?

b) What is the value of the common ratio r?

c) Write down the next three terms.

d) Give the value of n for the term which is -1.

e) Find the tenth term.

5) A population of bacteria doubles every 20 minutes. The initial population of bacteria is 1000.

a) How many bacteria will there be in 2 hours?

b) When will the population be 512,000?

Answers: 1)a) 9, 27, 81, b) 2, 1/4, 1/32, c) 4, 4√2, 8, d) 12, -6, 3, 2)a) 5, 10, 20, b) 8, 4, 2, c) 512, -128, 32, d) -1/27, -1/3, -3, 3)a) 1, 2, 4, b) 25, 5, 1, c) -3, 9, -27, d) 1/2, 2, 8, 4)a) -729, b) -1/3, c) -9, 3, -1, d) 7, e) 1/27, 5)a) 64,000, b) 3 hours.