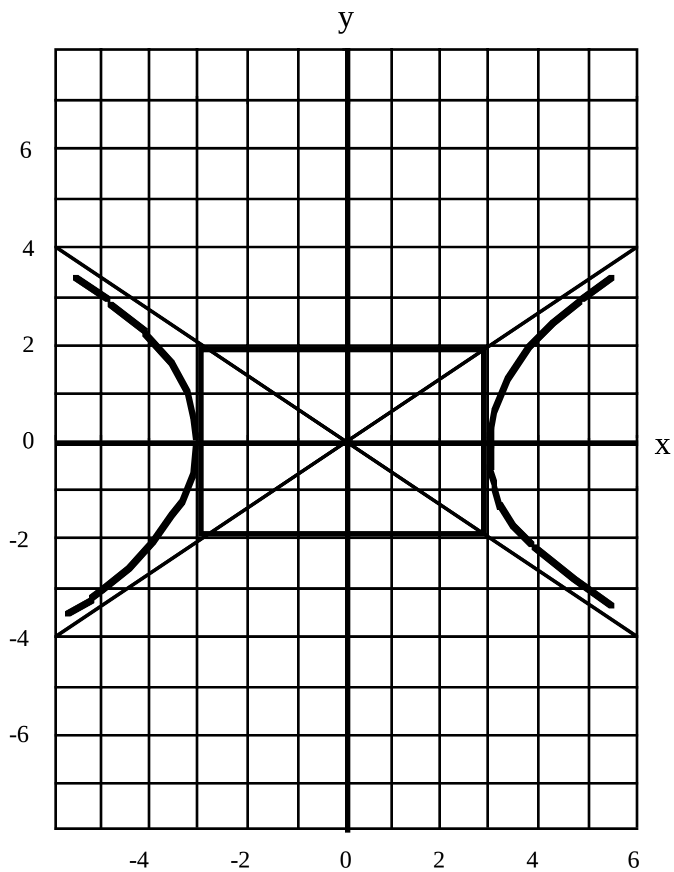
Conics : The Hyperbola 40

The hyperbola is a conic with two branches. It has the following two standard forms when centered at the origin.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

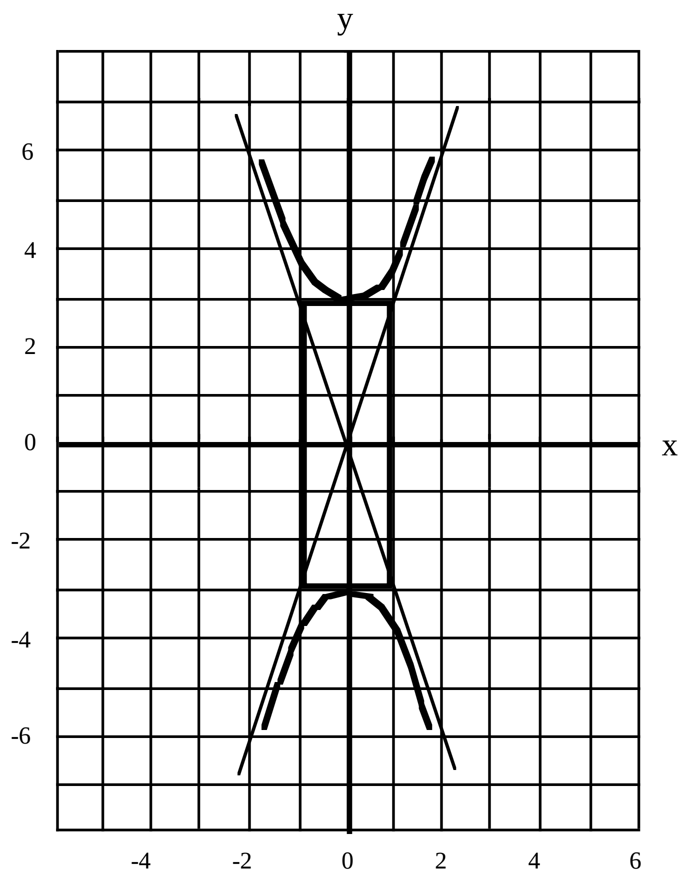
We can use a table of values to graph the equation, but it is much simpler to use the following method. A rectangle is drawn first. It has the dimensions, 2a by 2b. The asymptotes are straight lines drawn through the corners as shown in the example below. The branches approach, but never touch the asymptotes. The hyperbola, with vertices at (±3, 0) is shown below. It has the equation;

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$



Another example, with the vertices on the y-axis at (0, ±3), is shown below. It has the equation;

$$\frac{y^2}{9} - \frac{x^2}{1} = 1$$



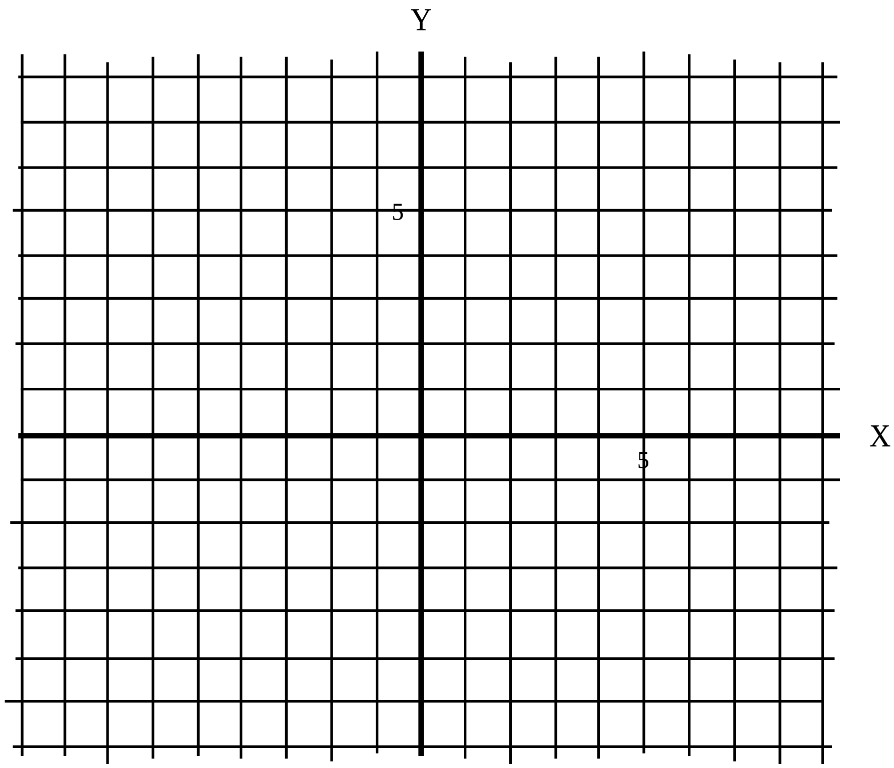
1) Graph the equations for each hyperbola given below. Show the “box” and the asymptotes.

a) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

b) $\frac{y^2}{16} - \frac{x^2}{9} = 1$

c) $\frac{(x-2)^2}{4} - \frac{(y+3)^2}{4} = 1$

d) $\frac{(y-1)^2}{4} - \frac{(x+4)^2}{1} = 1$



2) Find the coordinates of the center and vertices for each hyperbola in question number one.

3) Give the general form for each of the equations given in question number one.

4) Find the standard form for each of the following general forms.

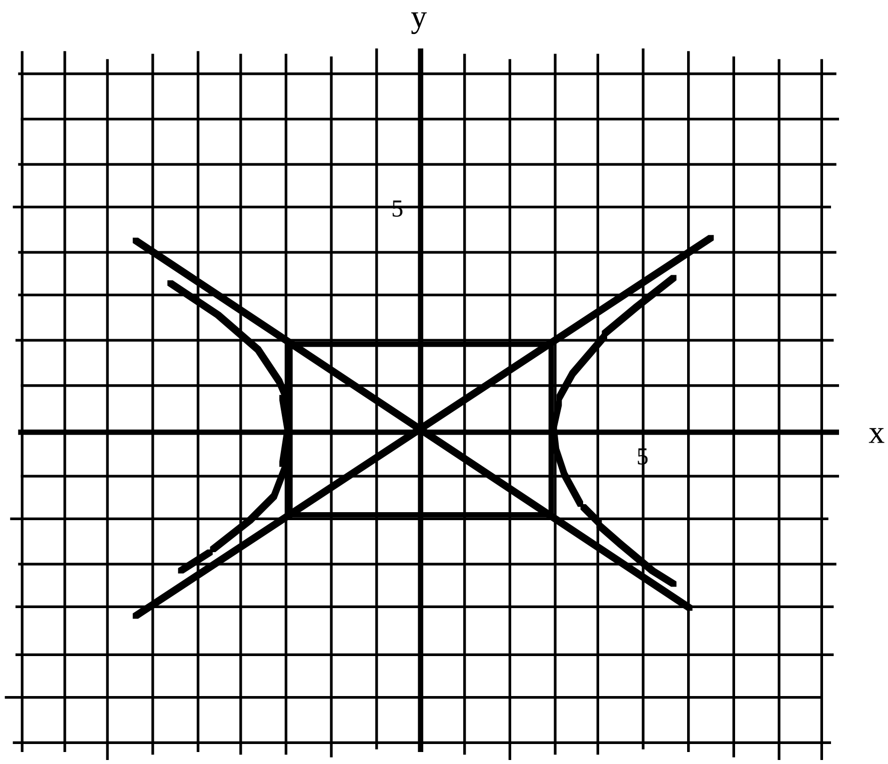
a) $4x^2 - 16y^2 = 64$

b) $9x^2 + 36x - 4y^2 = 0$

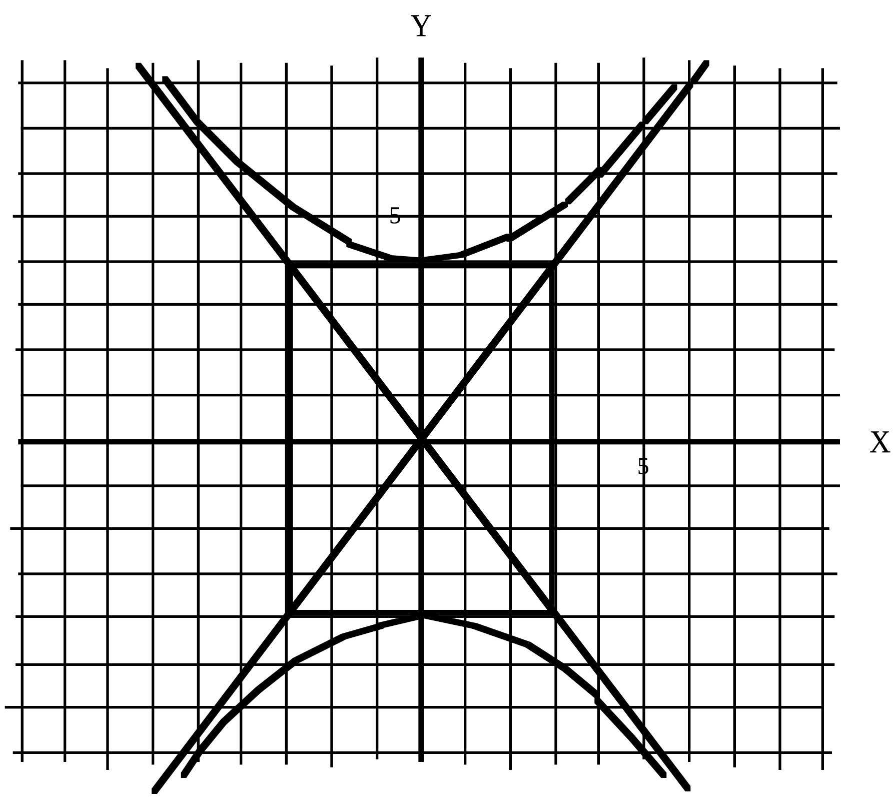
c) $9y^2 - x^2 + 10x - 34 = 0$

d) $4x^2 - 25y^2 + 16x + 150y - 309 = 0$

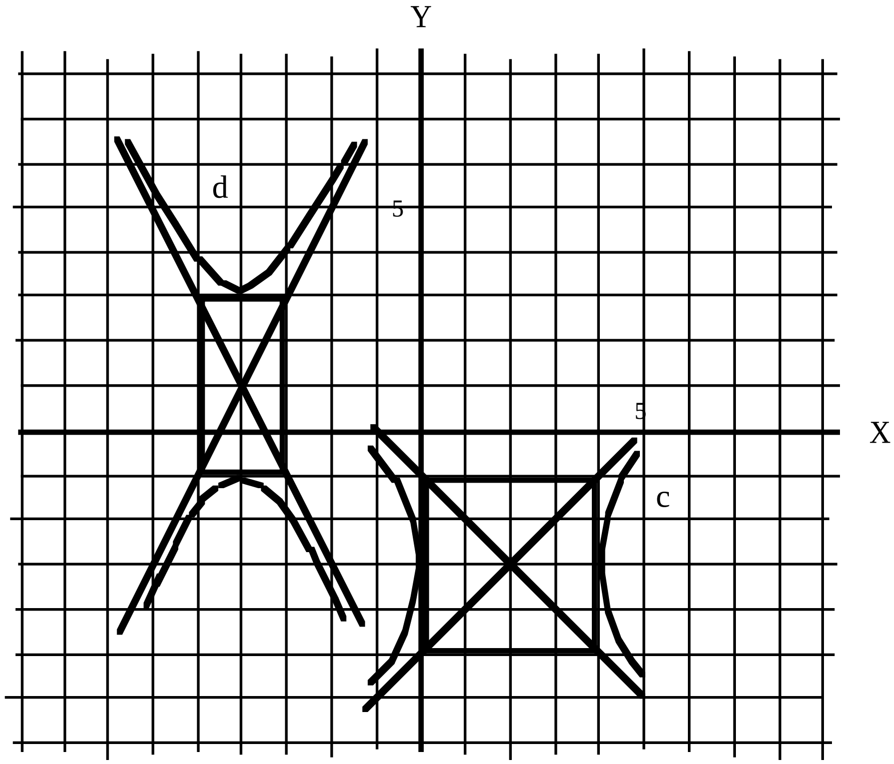
Answers: 1)a),



b),



c), d),



2)a), c; (0, 0); v; (±3, 0), b) c; (0, 0); v; (0, ±4),
c) c; (2, -3); v; (0, -3) and (4, -3), d) c; (-4, 1); v; (-4, 3) and (-4, -1),

3)a) $4x^2 - 9y^2 - 36 = 0$, b) $9y^2 - 16x^2 - 144 = 0$,
c) $x^2 - y^2 - 4x - 6y - 9 = 0$, d) $y^2 - 4x^2 - 2y - 32x - 67 = 0$,

4)

a) $\frac{x^2}{16} - \frac{y^2}{4} = 1$, b) $\frac{(x+2)^2}{4} - \frac{y^2}{9} = 1$

c) $\frac{y^2}{1} - \frac{(x-5)^2}{9} = 1$ d) $\frac{(x+2)^2}{25} - \frac{(y-3)^2}{4} = 1$