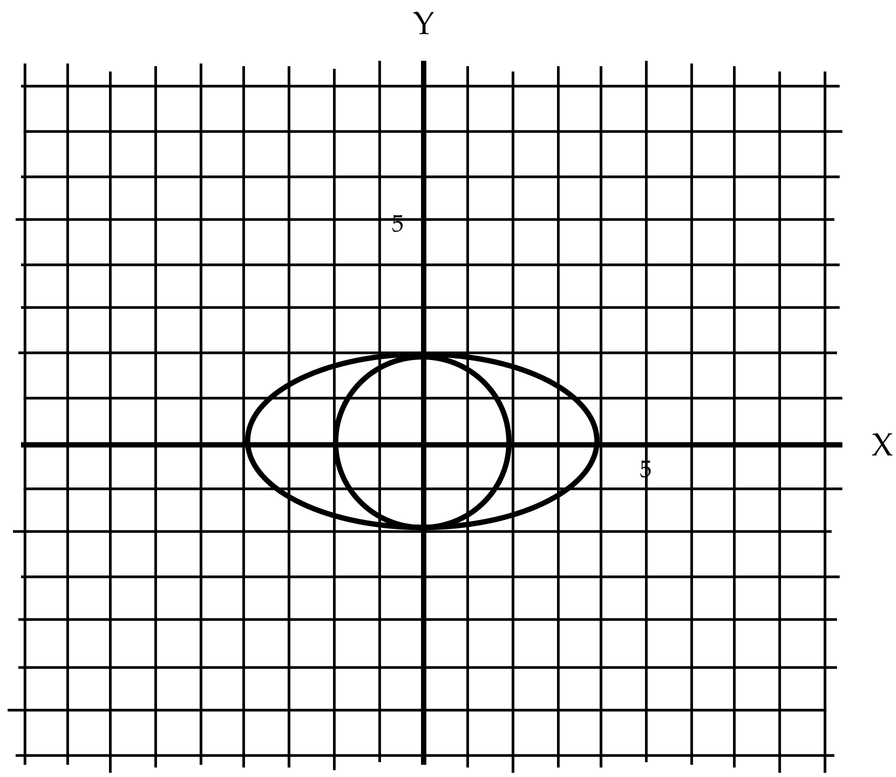
Conics : Ellipses 30

The graph of the equation; $x^2 + y^2 = 4$, is a circle with a radius of 2 which is centered at the origin. If we divide x by 2, the new equation is; $(x/2)^2 + y^2 = 4$, which can be written as,

$$\frac{x^2}{4} + y^2 = 4$$

This is the equation of an ellipse. The circle is stretched by a factor of two in the x direction. The two graphs are shown below.



The equation above may be rewritten.

$$\frac{x^2}{4} + y^2 = 4 \quad \text{->} \quad x^2 + 4y^2 - 16 = 0$$

This equation is the **general form**. The right side equals zero.

We can also write the equation in another form.

$$\frac{x^2}{4} + y^2 = 4 \quad \text{->} \quad \frac{x^2}{16} + \frac{y^2}{4} = 1$$

This is the **standard form**. The right side always equals one.

The general form can be converted to the standard form by completing the square.

Example:

$4x^2 + y^2 + 24x - 4y + 36 = 0$ ->

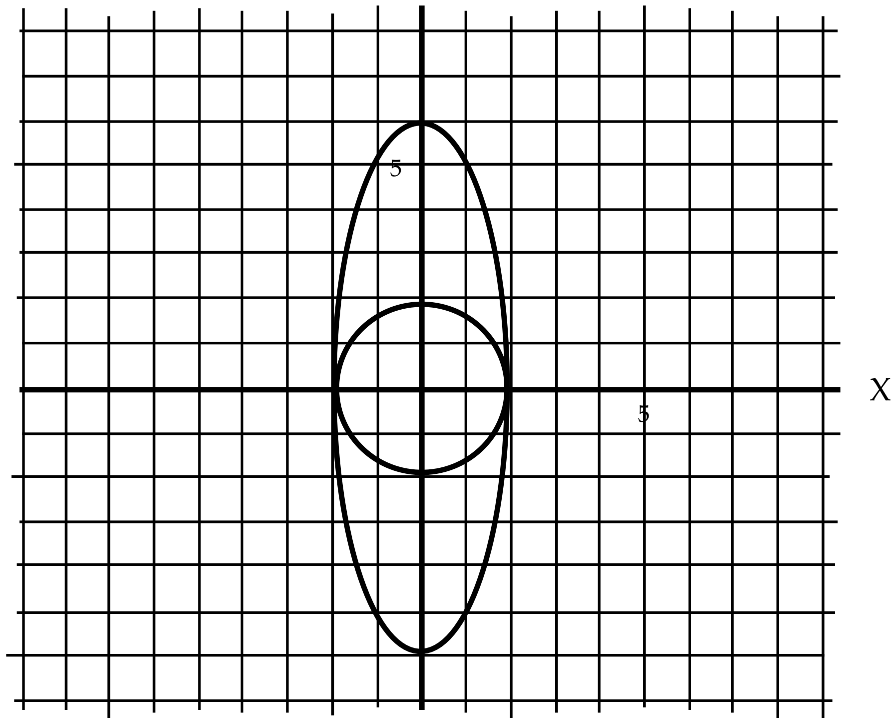
$4(x^2 + 6x + \_\_\_) + (y^2 - 4y + \_\_\_) = -36$ ->

$4(x + 3)^2 + (y - 2)^2 = -36 + 36 + 4$ ->

$$\frac{(x+3)^2}{1} + \frac{(y-2)^2}{4} = 1$$

The circle: $x^2 + y^2 = 4$, when stretched by a factor of 3 in the y direction, has the equation; $x^2 + (y/3)^2 = 4$. This equation can be written in standard form.

$$\frac{x^2}{4} + \frac{y^2}{36} = 1$$

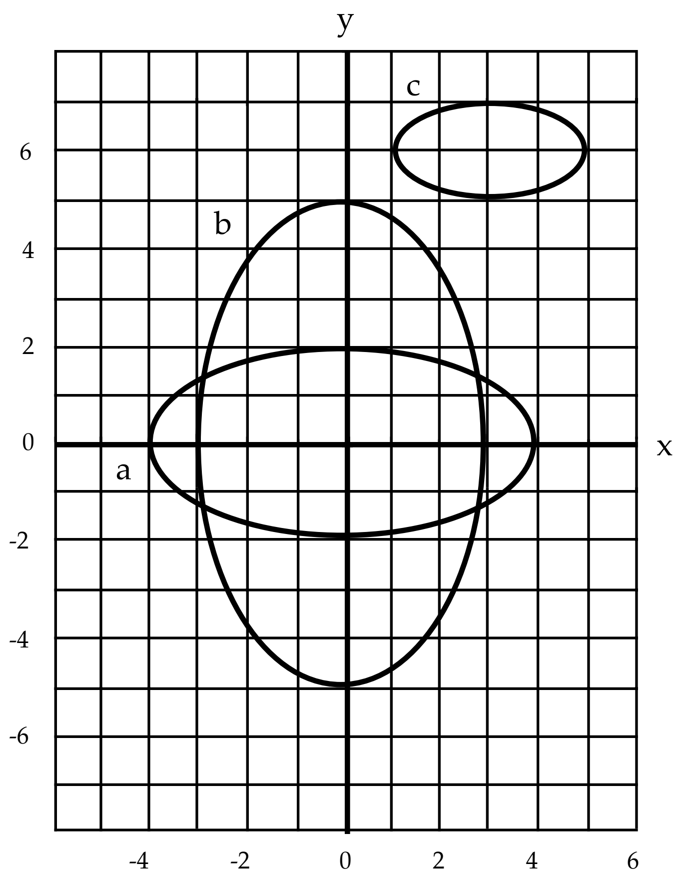


In the equation of an ellipse, the coefficients of x^2 and y^2 are not equal.

We can shift the ellipse in the usual way. The equation for the above ellipse when it has been shifted right by 2 and down by 3 is:

$$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{36} = 1$$

Example: The graphs of three ellipses are shown below.



The equations for the above ellipses are:

a) $\frac{x^2}{16} + \frac{y^2}{4} = 1$

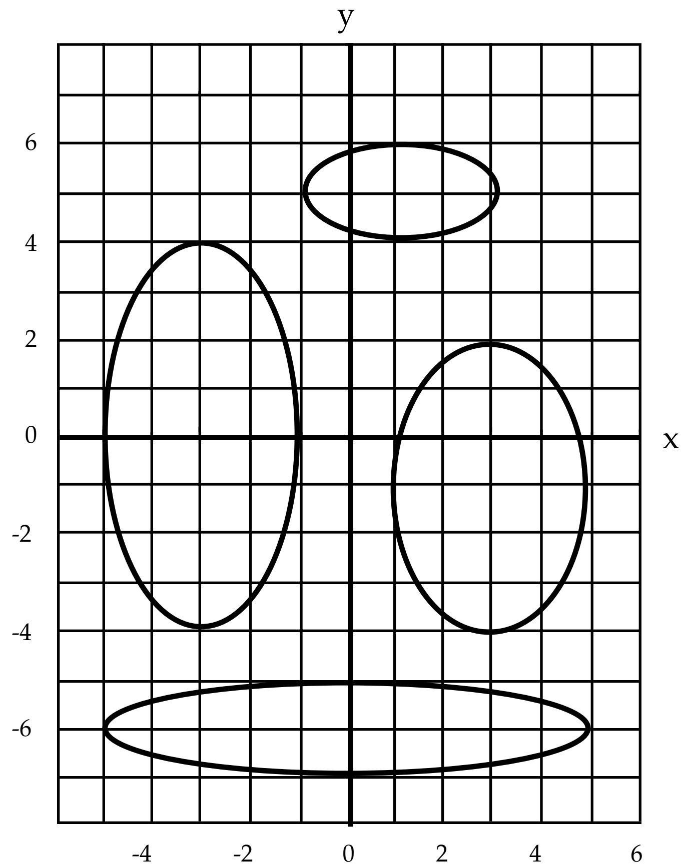
b) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

c) $\frac{(x-3)^2}{4} + \frac{(y-6)^2}{1} = 1$

Questions:

1) Convert the three equations from the above example into general form.

2) Write down the equations for the graphs shown below.



3) Transform the equations below into standard form.

a) $9x^2 + 4y^2 - 36 = 0$

b) $16x^2 + 25y^2 - 400 = 0$

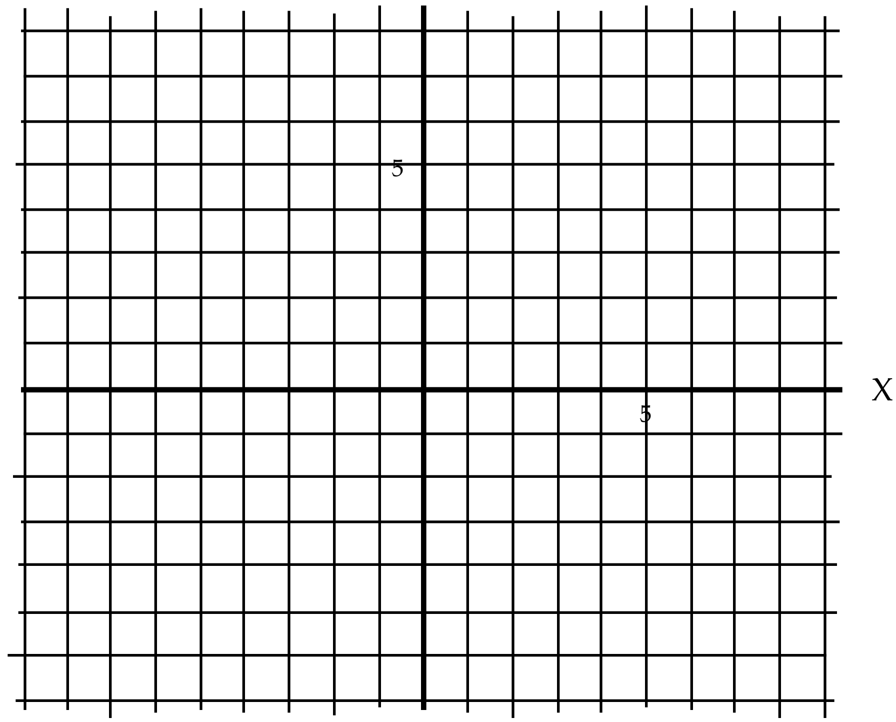
c) $4x^2 + 16y^2 - 32y = 0$

d) $25x^2 + 4y^2 - 150x + 125 = 0$

e) $4x^2 + 9y^2 - 8x + 36y + 4 = 0$

f) $25x^2 + 9y^2 + 250x - 54y + 481 = 0$

4) Graph the equations; a), b) and f) from question 3.



Answers:

1)a) $x^2 + 4y^2 - 16 = 0$, b) $25x^2 + 9y^2 - 225 = 0$,
c) $x^2 + 4y^2 - 6x - 48y + 149 = 0$, 2)

a) $\frac{(x-3)^2}{4} + \frac{y^2}{16} = 1$, b) $\frac{(x-1)^2}{4} + \frac{(y-5)^2}{1} = 1$

c) $\frac{x^2}{25} + \frac{(y-6)^2}{1} = 1$ d) $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{9} = 1$

3)

a) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ b) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

c) $$\frac{x^2}{4}+\frac{(y-1)^2}{1}=1$$

d) $$\frac{(x-3)^2}{4}+\frac{y^2}{25}=1$$

e) $$\frac{(x-1)^2}{9}+\frac{(y+2)^2}{4}=1$$

f) $$\frac{(x+5)^2}{9}+\frac{(y-3)^2}{25}=1$$

4)