

Conics : Circles 20

When we find a table of values for the relation; $x^2 + y^2 = 25$, and graph them, we find that the relation is a circle with a radius of 5, which is centered at the origin.

In general, the relation; $(x - h)^2 + (y - k)^2 = R^2$, is a circle with a radius of R, which is centered at the point (h , k).

The equation;

$$(x - 2)^2 + (y + 4)^2 = 16 \quad (\text{standard form})$$

can be converted to its general form by expanding.

$$(x - 2)^2 + (y + 4)^2 = 16 \rightarrow x^2 - 4x + 4 + y^2 + 8y + 16 = 16 \rightarrow$$

$$x^2 + y^2 - 4x + 8y + 4 = 0 \quad (\text{general form})$$

The general form can be converted to the standard form.

$$x^2 + y^2 + 8x - 6y - 11 = 0 \quad (\text{general form})$$

$$\rightarrow (x^2 + 8x + \underline{\quad}) + (y^2 - 6y + \underline{\quad}) - 11 = 0$$

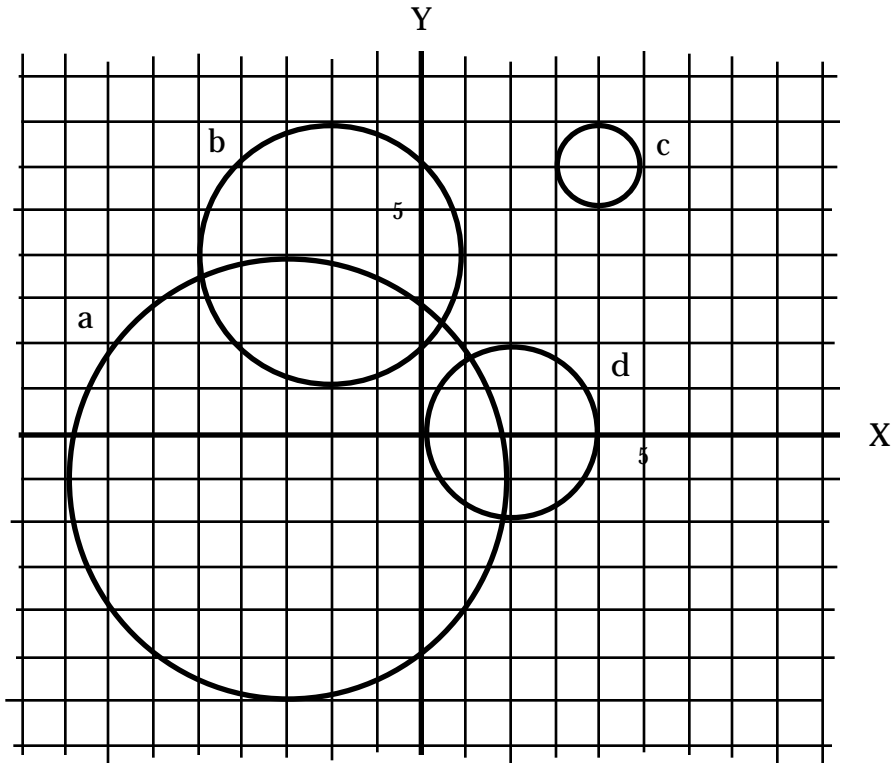
$$\rightarrow (x^2 + 8x + 16) + (y^2 - 6y + 9) - 11 = + 16 + 9$$

$$\rightarrow (x + 4)^2 + (y - 3)^2 = 36 \quad (\text{standard form})$$

Examples of graphs of circles are shown below.

a) $(x + 3)^2 + (y + 1)^2 = 25$ b) $(x + 2)^2 + (y - 4)^2 = 9$

c) $(x - 4)^2 + (y - 6)^2 = 1$ d) $(x - 2)^2 + y^2 = 4$

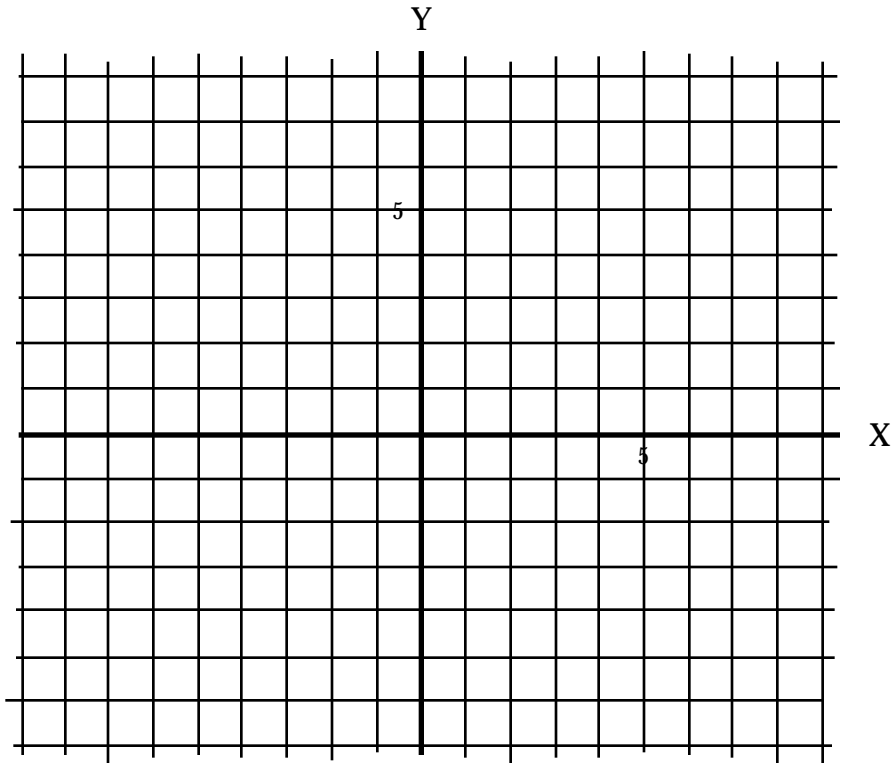


1) Give the general form for each of the above equations.

2) Graph the following equations.

a) $(x + 4)^2 + (y - 5)^2 = 4$

b) $(x - 2)^2 + (y + 3)^2 = 16$



3) Give the coordinates of the center of the circle and give the radius.

a) $x^2 + y^2 = 16$

b) $x^2 + y^2 = 8$

c) $(x - 2)^2 + y^2 = 25$

d) $(x + 4)^2 + (y - 5)^2 = 4$

e) $3x^2 + 3y^2 = 27$

f) $4x^2 + 4(y + 3)^2 = 9$

4) Find the equation of each circle.

a) radius = 6, center = (5, -3)

b) radius = $\sqrt{5}$, center = (0, 2)

c) radius = 3, circle is tangent to x axis and y axis.

5) Convert to standard form.

a) $x^2 + y^2 - 4y = 0$

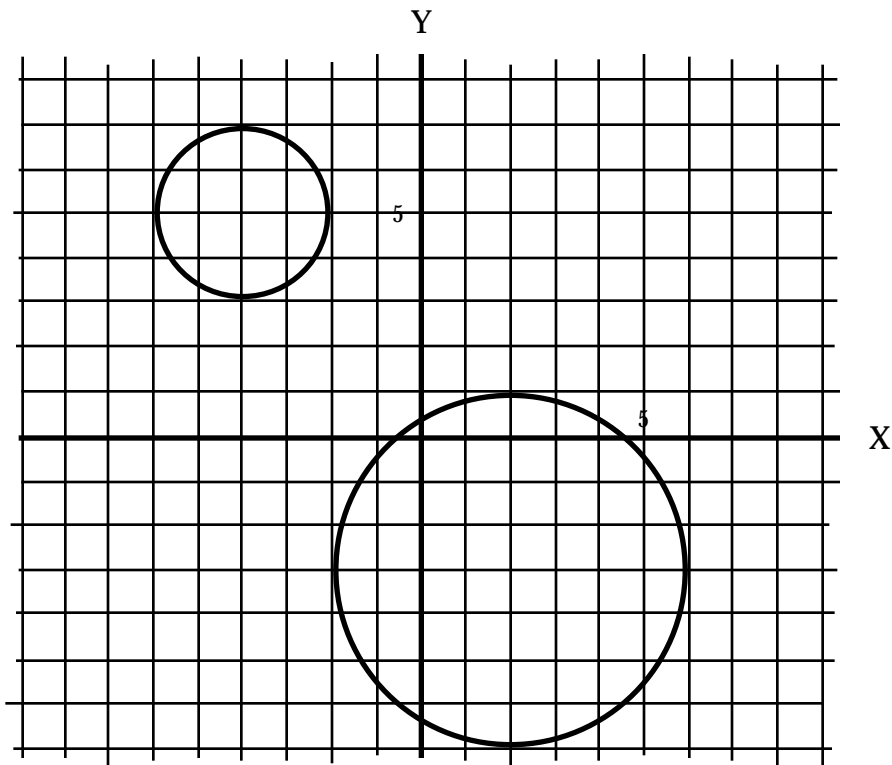
b) $x^2 + y^2 + 2x = 0$

c) $x^2 + y^2 + 4x + 2y - 4 = 0$

d) $x^2 + y^2 - 10x + 4y + 25 = 0$

e) $2x^2 + 2y^2 + 8x + 40y = 0$

Answers: 1)a) $x^2 + y^2 + 6x + 2y - 15 = 0$, b) $x^2 + y^2 + 4x - 8y + 11 = 0$,
 c) $x^2 + y^2 - 8x - 12y + 51 = 0$, d) $x^2 - 4x + y^2 = 0$, 2)



3)a) $(0, 0)$; 4, b) $(0, 0)$; $2\sqrt{2}$, c) $(2, 0)$; 5, d) $(-4, 5)$; 2, e) $(0, 0)$; 3, f) $(0, -3)$; $3/2$, 4)a) $(x - 5)^2 + (y + 3)^2 = 36$, b) $x^2 + (y - 2)^2 = 5$,
 c) $(x \pm 3)^2 + (y \pm 3)^2 = 9$, 5)a) $x^2 + (y - 2)^2 = 4$, b) $(x + 1)^2 + y^2 = 1$, c) $(x + 2)^2 + (y + 1)^2 = 9$, d) $(x - 5)^2 + (y + 2)^2 = 4$,
 e) $(x + 2)^2 + (y + 10)^2 = 104$.