The Binomial Theorem-60

The following are examples of binomials.

(x + y), (a + b), (2x - 3y), and so on...

An important problem in mathematics is finding the expansion of the power of a binomial. Some examples are shown below.

$(x + y)^0 = 1$

$(x + y)^1 = 1x + 1y$

$(x + y)^2 = 1x^2 + 2xy + 1y^2$

$(x + y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$

$(x + y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

etc...

In general we have the **Binomial Theorem**:

$(x + y)^n = {}_nC_0\, x^n + {}_nC_1\, x^{n-1}y + {}_nC_2\, x^{n-2}y^2 + \ldots + {}_nC_n\, y^n$

The coefficients of the terms can be found by using the **combination formula**.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

The coefficients can also be found by using **Pascal's Triangle**.

Each term in the triangle can be found by adding the two terms above it. The first four rows of the Pascal Triangle are shown below.

1 1

1 2 1

1 3 3 1

1 4 6 4 1

The nth row contains the coefficients of the terms of the expansion of $(x + y)^n$.

Examples:

$(x - y)^4 = (x + (-y))^4 = 1x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + 1y^4$

$(a + 2b)^3 = 1a^3 + 3a^2(2b) + 3a(2b)^2 + 1(2b)^3 = 1a^3 + 6a^2b + 12ab^2 + 8b^3$

Problems:

1)a) Write down the sixth row of Pascal's triangle.

b) Expand $(a + b)^6$

2) In the expansion of $(x + y)^9$, give the coefficients (numbers only) for the first three terms.

3) Expand the following:

a) $(a + b)^4$

b) $(u - 3v)^3$

c) $(3x + 2y)^5$

d) $(y/2 + 1)^4$

4) Give the fifth term in the expansion of $(x + y)^{12}$.

Answers: 1)a) 1, 6, 15, 20, 15, 6, 1, b) $a^6 + 6ab^5 + 15a^2b^4 + 20a^3b^3 + 15a^4b^2 + 6a^5b + b^6$, 2) The first three coefficients are ${}_9C_0$, ${}_9C_1$, ${}_9C_2$, or 1, 9, and 36, 3)a) $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$, b) $u^3 - 9u^2v +$

$27uv^2 - 27v^3$, c) $243x^5 + 810x^4y + 1080x^3y^2 + 720x^2y^3 + 240xy^4 + 32y^5$, d) $y^4/16 + y^3/2 + 3/2\ y^2 + 2y + 1$, 4) ${}_{12}C_4\ x^8\ y^4 = 495\ x^8\ y^4$.