

Permutations 2-40

When dealing with permutations of objects, sometimes we have to deal with a collection of objects in which some are alike.

Example 1:

Suppose that we have the following collection of objects; A, B, C, C, and C. There are five objects, three of which are indistinguishable. The total number of permutations is $5!$, if we can distinguish between the C's.

But the C's can be ordered in $3!$ ways. So the true number of permutations is $5!/3! = 20$, if the C's are indistinguishable.

Example 2:

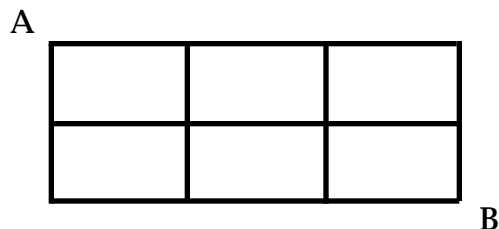
Suppose that we have nine objects; X, X, X, X, Y, Y, Y, Y, and Y.

If the objects were all distinguishable, there would be $9!$ permutations. But there are four indistinguishable X's and five indistinguishable Y's.

Therefore, the true number of permutations is $9!/(4! \times 5!) = 126$.

Example 3:

Suppose that a person can walk from A to B, moving only down (D) or right (R) across the grid. How many routes are there to move from A to B?



The routes can be DDRRR, RDRDR, RRRDD and so on. The total number of routes is $5!/(2! \times 3!) = 10$. Try to find all of these routes.

Questions:

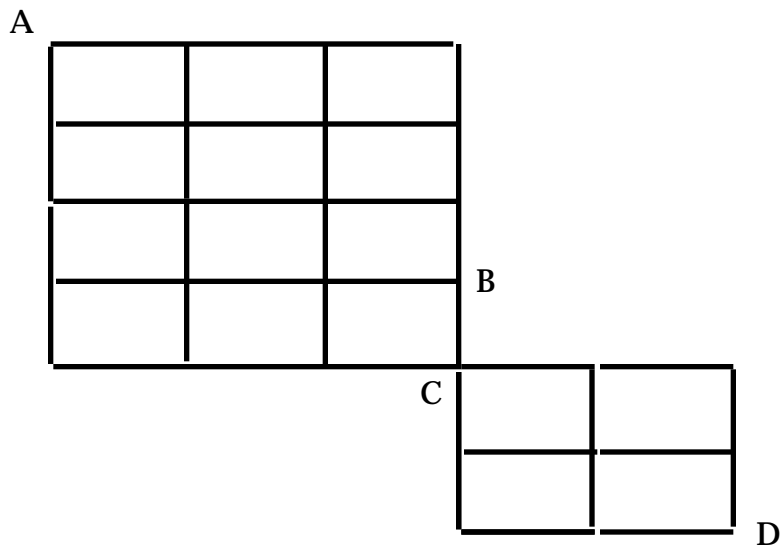
1) Evaluate: $12!/(5! \times 7!)$

2)a) How many permutations are there for the letters in the word CANADA?

b) How many different words can be made using all of the letters in the word STATISTICS.

3) A multiple choice quiz has nine questions. The choices are A, B, and C. The answer key has 3 A's, 3 B's and 3 C's. How many different answer keys are possible?

4) Answer the questions below based on the following grid. Assume that you are only allowed to move right (R) or down (D).



a) How many paths are there between A and B?

b) How many paths are there between A and C?

c) How many paths are there between A and D?

Answers: 1) 792, 2)a) $6!/(3! \times 3!) = 120$, b) $10!/(3! \times 3! \times 2!) = 50,400$, 3) $9!/(3! \times 3! \times 3!) = 1680$, 4)a) $6!/(3! \times 3!) = 20$, b) $7!(4! \times 3!) = 35$, c) $7!/(4! \times 3!) \times 4!/(2! \times 2!) = 210$.