

Counting Problems 15

Examples:

1)a) How many two digit numbers are there? b) How many of these contain no 5? c) How many two digit numbers contain at least one 5?

Answer: 1)a) $9 \times 10 = 90$, b) $8 \times 9 = 72$, c) $90 - 72 = 18$

2) A pair of dice are rolled.

a) How many possible outcomes are there? b) How many outcomes do not have a three? c) How many outcomes have at least one three?

Answer: 2)a) $6 \times 6 = 36$, b) $5 \times 5 = 25$, c) $36 - 25 = 11$.

3) A multiple choice test has five questions. Each question has four possible answers; a, b, c, or d. Find the number of possible answer keys.

Answer: 3) $N = 4 \times 4 \times 4 \times 4 \times 4 = 1024$.

4) Three marbles, red, green, and blue, are placed in two boxes. How many different ways can this be done?

Answer: 4) There are two places for each marble. $2 \times 2 \times 2 = 8$.

For the following problems, show your reasoning when giving the answer.

1)a) How many three digit numbers are there? (first digit $\neq 0$)

b) How many of these are divisible by five?

c) How many of these are greater than 699?

- d) How many of these do not contain a four?
- e) How many of these contain at least one four?
- f) How many of these contain two fours?
- g) How many contain only one four?

2)a) A restaurant serves 5 types of sandwiches, 3 types of soup, and 2 deserts. If a meal consists of a sandwich, a bowl of soup, and a desert, how many meals are there?

b) How many meals are there, if a meal consists of 2 different sandwiches, a bowl of soup, and a desert? (Watch out!)

3) A true - false quiz has 6 questions. How many possible answer keys are there?

4)a) A combination lock requires 3 numbers from 0 to 59 to open it. How many combinations are there?

b) How many combinations are there if the second number must be different from the first?

5) There are three towns, A, B, and C. There are 3 roads from A to B. There are 4 roads from B to C.

a) How many different routes are there from A to C?

b) How many different routes are there from A to C and back to A?

c) How many different routes are there from A to C and back to A, if each road can only be traveled on once?

6) A computer needs to be accessed by a code. The code consists of 3 different letters followed by 2 different digits. How many different codes are there?

7) Three dice are rolled.

a) How many outcomes are there?

b) How many of these do not have a 2 or a 5?

c) How many of these have at least one 2, or one 5?

8) Words can be made using four letters.

a) How many words can be made if repetitions are allowed?

b) How many words can be made if repetitions are not allowed?

9) Five letters are placed in three mailboxes. How many ways can this be done?

10) At a club, there are 4 candidates for president, 3 for vice-president, and 2 for secretary. How many possibilities are there?

11) How many five digit numbers contain only 3's and 4's.

12) Three dice are rolled, and the sum is calculated. How many ways can a sum of 8 be obtained?

Answers: 1)a) $9 \times 10 \times 10 = 900$, b) $9 \times 10 \times 2 = 180$, c) $3 \times 10 \times 10 = 300$, d) $8 \times 9 \times 9 = 648$, e) $900 - 648 = 252$, f) 26, g) 225, 2)a) $5 \times 3 \times 2 = 30$, b) $\{(5 \times 4) / 2\} \times 3 \times 2 = 60$ (The order doesn't matter for the sandwiches!), 3) $2 \times 2 \times 2 \times 2 \times 2 = 64$, 4)a) $60 \times 60 \times 60 = 216,000$, b) $60 \times 59 \times 60 = 212,400$,

5)a) $3 \times 4 = 12$, b) $3 \times 4 \times 4 \times 3 = 144$, c) $3 \times 4 \times 3 \times 2 = 72$, 6) $26 \times 25 \times 24 \times 10 \times 9 = 1,404,000$, 7)a) $6 \times 6 \times 6 = 216$, b) $4 \times 4 \times 4 = 64$, c) $216 - 64 = 152$, 8)a) $26 \times 26 \times 26 \times 26 = 456,976$, b) $26 \times 25 \times 24 \times 23 = 358,800$, 9) $3 \times 3 \times 3 \times 3 \times 3 = 243$, 10) $4 \times 3 \times 2 = 24$, 11) $2 \times 2 \times 2 \times 2 \times 2 = 32$, 12) 21; 116, 161, 611, 224, ... etc.