

Combinations W.S. 55 (Order is not Important)

Examples:

Combinations Problem

A tennis team of four members is chosen from a group of 21. How many possibilities are there?

The number of possibilities is $= {}_{21}C_4 = 5985$.

Handshake Problem

How many handshakes are there, if all people in a group of five, shake everyone else's hand in the group once?

The number of handshakes $= {}_5C_2 = 10$

Number of combinations of n objects taken 1, or 2, or 3 or n, at a time

$$N = 2^n - 1$$

How many sums of money can be made with a penny, a nickel, and a dime?

$$N = 2^3 - 1 = 7$$

Five cards are dealt from a deck of 52. Find the number of hands:

- | | |
|-------------------------------|--|
| a) total number of hands | ${}_{52}C_5$ |
| b) all cards are hearts | ${}_{13}C_5$ |
| c) two kings | ${}_4C_2 \times {}_{48}C_3$ |
| d) two kings and three queens | ${}_4C_2 \times {}_4C_3$ |
| e) at least one ace | ${}_{52}C_5 - {}_{48}C_5$ (total - hands with no aces) |

Problems

1) Solve for n .

a) ${}^7C_3 = {}^nC_4$ b) ${}^nC_5 = {}^nC_8$ c) ${}^nC_{n-2} = 45$

2) Find the number of handshakes at a party of 15, if each person shakes the hand of everyone else once.

3) A committee of 3 is chosen from a group of 30 people. How many possible committees are there?

4) A teacher assigns 5 problems out of a set of 10 problems from one text, and 3 problems out of a set of 8 from another text. How many possible assignments are there?

5) A poker hand consists of 5 cards dealt from a deck of 52 cards. Find the number of possibilities for hands that contain:

a) no hearts.

b) five red cards.

c) exactly two clubs.

d) two clubs and three hearts.

e) at least 3 aces.

6) Eight boys and six girls are in a chess club. Four of these students are chosen to be on the chess team.

a) How many possibilities are there?

b) How many possibilities are there, if 2 boys and 2 girls must be on the team.

7) How many sums of money can be made with a quarter, a dime, a nickel, and a penny?

8) Find the number of a) combinations, and b) permutations, using four letters, from the word CANADA.

Answers: 1)a) 7, b) 13, c) 10, 2) ${}_{15}C_2 = 105$, 3) ${}_{30}C_3 = 4060$, 4) ${}_{10}C_5 \times {}_8C_3 = 14,112$, 5)a) ${}_{39}C_5 = 575,757$, b) ${}_{26}C_5 = 65,780$, c) ${}_{13}C_2 \times {}_{39}C_3 = 712,842$, d) ${}_{13}C_2 \times {}_{13}C_3 = 22,308$, e) ${}_4C_3 \times {}_{48}C_2 + {}_4C_4 \times {}_{48}C_1 = 4,560$, 6)a) ${}_{14}C_4 = 1001$, b) ${}_8C_2 \times {}_6C_2 = 420$, 7) $2^4 - 1 = 15$, 8)a) 7, b) 72.